Vocabulary for Linear Algebra

Below is the list of (hopefully) all terms that students of linear algebra course are responsible for learning and understanding. The list was mostly compiled by Dr. Curtin on the basis of the textbook, with some definitions added and some removed. I have rephrased the opening of each definition in the form you might expect on a quiz or exam. I leave it to you to complete most definition as practice.

(1) A *matrix* is (Page 9)
(2) An $n \times m$ *matrix* is a matrix with n rows and m columns. (Page 9)
(3) A matrix A is called *square* if (Page 9)
(4) The *diagonal* of a marix $A = (a_{ij})$ consists of (Page 9)
(5) A square matrix A is called *diagonal* if (Page 9)
(6) A square matrix A is called *upper triangular* if (Page 10)
(7) A square matrix A is called *lower triangular* if (Page 10)
(8) The *zero matrix* is the matrix (Page 10)
(9) A *column vector* is (Page 10)
(10) The *coefficient matrix* of a system of linear equations

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1m}x_m &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2m}x_m &= b_2 \\ \vdots &= \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nm}x_m &= b_n \end{aligned}$$

is (Page 11)

(11) The *augmented matrix* of a system of linear equations

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1m}x_m &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2m}x_m &= b_2 \\ \vdots &= \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nm}x_m &= b_n \end{aligned}$$

is (Page 11)

(12) A *leading (pivot) entry* in a row of a matrix is the leftmost non-zero entry in this row.
(13) A matrix is said to be in *row-echelon form* if it satisfies the following condition:
   - if a row has a leading non-zero entry, then each row above it contains a leading non-zero entry further to the left.

(14) A matrix is said to be in *reduced row-echelon form* if it satisfies the following three conditions:
   - it is in row-echelon form;
   - all leading entries are equal to 1;
   - each column with a leading entry has only one non-zero entry.

   (compare to (Page 16))

(15) A *pivot column* in a matrix A is a column of A that will contain a leading entry after A is reduced to row-echelon form.
(16) A *leading (or basic) variable* in a system of linear equations is a variable that corresponds to a pivot column of its coefficient matrix.
(17) A *free variable* in a system of linear equations is a variable that corresponds to a non-pivot column of its coefficient matrix.
(18) The following three operations on a matrix A are called *elementary row operations* (Page 16)
(19) A system of equations is said to be *consistent* if (thm 1.3.1)
(20) A system of equations is said to be *inconsistent* if (thm 1.3.1)
(21) The *rank* of a matrix A is the number of pivot columns in A (= number of leading entries in $ref(A)$. (compare to def 1.3.2)
(22) The *sum* of two matrices $A = (a_{ij})_{nm}$ and $B = (b_{ij})_{nm}$ of the same size is the matrix with entries (def 1.3.5)

(23) The *product* of a scalar k and a matrix $A = (a_{ij})_{nm}$ is the matrix with entries (def 1.3.5)
(24) Consider vectors $\vec{v} = (v_i)_n$ and $\vec{w} = (w_i)_n$ of length n. The dot product of $\vec{v} \cdots \vec{w}$ is (def 1.3.6)
(25) Given a matrix $A = (a_{ij})_{nm}$ and a vector $\vec{v} = (v_i)_m$, the vector $A\vec{v}$ has i^{th} component (def 1.3.7)
(26) A vector $\vec{b} \in \mathbb{R}^n$ is called a *linear combination* of vectors $\vec{v_1}$, $\vec{v_2}$, $\ldots \vec{v_m} \in \mathbb{R}^n$ if (def 1.3.9)
(27) A function T from $\mathbb{R}^m$ to $\mathbb{R}^n$ is called a *linear transformation* if (def 2.1.1)
(28) The $n \times n$ *identity matrix* is the matrix (Page 46)
(29) The *standard vectors* $e_1, \ldots e_n$ of $\mathbb{R}^n$ are the vectors (Page 48)
(30) Two vector $\vec{x}$ and $\vec{y}$ in $\mathbb{R}^n$ are *parallel* if (def A.3)
(31) The *length* $||\vec{x}||$ of a vector $\vec{x}$ in $\mathbb{R}^n$ is (def A.6)
(32) A vector $\vec{u}$ in $\mathbb{R}^n$ is a called a *unit vector* if (def A.7)
(33) Two vector $\vec{x}$ and $\vec{y}$ in $\mathbb{R}^n$ are *orthogonal* if (def A.8)
(34) Given a line L and a vector $\vec{x}$ in $\mathbb{R}^2$, write $\vec{x} = \vec{x}^{||} + \vec{x}^{\perp}$, where $\vec{x}^{||}$ is parallel to L and $\vec{x}^{\perp}$ is perpendicular to L. The *orthogonal projection* of $\vec{x}$ onto L is the transformation (def 2.2.1)
(35) Given a line L through the origin and a vector $\vec{x}$ in $\mathbb{R}^2$, write $\vec{x} = \vec{x}^{||} + \vec{x}^{\perp}$, where $\vec{x}^{||}$ is parallel to L and $\vec{x}^{\perp}$ is perpendicular to L. The *reflection* of $\vec{x}$ about L is the transformation (def 2.2.1)
(36) Let B be an $n \times p$ matrix and let A be a $p \times m$ matrix. Then the *product* BA is the matrix of (def 2.3.1)
(37) Two matrices A and B *commute* whenever (thm 2.3.3)
(38) Let B be an $n \times p$ matrix and let A be a $p \times m$ matrix. Then the *ijth entry of BA* is (thm 2.3.4)
(39) A square matrix A is said to be *invertible* if (def 2.4.2)
(40) The *span* of vectors $\vec{v}_1, \ldots, \vec{v}_m$ in $\mathbb{R}^n$ is
(41) The *image* of a function $f\colon X \to Y$ is (def 3.1.1)
(42) A function $f\colon X \to Y$ is said to be *one-to-one* if (Definition 3 in the handout at http://savchuk.myweb.usf.edu/teaching/2014A_math3105_linear_algebra/functions_handout)
(43) A function $f\colon X \to Y$ is said to be *onto* if (Definition 3 in the handout at http://savchuk.myweb.usf.edu/teaching/2014A_math3105_linear_algebra/functions_handout)
(44) A function $f\colon X \to Y$ is said to be *one-to-one correspondence* if (Definition 3 in the handout at http://savchuk.myweb.usf.edu/teaching/2014A_math3105_linear_algebra/functions_handout)
(45) The *identity function* (or *identity transformation*) *on a set* X is a function $id_X : X \to X$ defined by $id_X(x) = x$ for all $x \in X$.
(46) If $f\colon X \to Y$ is a one-to-one correspondence, a function $f^{-1}\colon Y \to X$ is called the *inverse to* f if (Definition 4 in the handout at http://savchuk.myweb.usf.edu/teaching/2014A_math3105_linear_algebra/functions_handout)
(47) A set of vectors V is said to be *closed under linear combinations* if (page 114)
(48) The *kernel* of a linear transformation $T(\vec{x}) = A\vec{x}$ is (def 3.1.5)

Many terms in Chapter 3 appear in a more general form in Chapter 4. I will list them in the general form, but students should know the special $\mathbb{R}^n$ form for quizzes on Chapter 3.

(49) A *linear relation* among vectors $\vec{v}_1, \ldots, \vec{v}_m$ in $\mathbb{R}^n$ is (def 3.2.6)
(50) A linear relation $c_1\vec{v}_1 + \cdots + \ldots + c_m\vec{v}_m = 0$ in $\mathbb{R}^n$ is *nontrivial* when (def 3.2.6)
(51) $n \times n$ matrices A and B are *similar* whenever (def 3.4.5)
(52) A *linear space* V is a set endowed with a rule for addition and scalar multiplication satisfying the following eight rules: (def 4.1.1)
(53) A subset W of a linear space V is called a *subspace* if (def 3.2.1, 4.1.2)
(54) Elements $f_1, f_2, \ldots, f_n$ *span* V if (def 4.1.3)
(55) Consider $f_1, f_2, \ldots, f_n$ in a linear space V. We say that f_i is *redundant* if (def 3.2.3, 4.1.3)
(56) Elements $f_1, f_2, \ldots, f_n$ of a linear space V are *linearly independent* if (def 3.2.3, 4.1.3)

(57) The coordinates of $f \in V$ with respect to a basis $\mathfrak{B} = (f_1, f_2, \ldots f_n)$ of V are (def 3.2.3,4.1.3)
(58) An element f of V is a *linear combination* of elements $f_1, f_2, \ldots, f_n$ if (above example 9 of 4.1)
(59) Elements $f_1, f_2, \ldots, f_n$ form a *basis* for V whenever (def 4.1.3)
(60) Given an ordered basis $\mathfrak{B} = (f_1, f_2, \ldots f_n)$ of a vector space V, the $\mathfrak{B}$-*coordinate vector of* f is (def 4.1.3)
(61) Given an ordered basis $\mathfrak{B} = (f_1, f_2, \ldots f_n)$ of a vector space V and $f \in V$, the $\mathfrak{B}$-*coordinate transformation of* f is (def 4.1.3)
(62) The *dimension* of a vector space V is (theorem 4.1.5)
(63) The standard basis of $\mathbb{R}^{2\times 2}$ is (example 15 of 5.1)
(64) The standard basis of P_n is (example 16 of 5.1)
(65) The standard basis of $\mathbb{C}$ of all complex numbers is (Exercises 5-40 (15 in particular) in Section 4.3)
(66) A vector space V is called *finite dimensional* if (Def 4.1.8)
(67) A vector space V is called *infinite dimensional* if (Def 4.1.8)
(68) Given linear spaces V and W, a function $T\colon V \to W$ is called a *linear transformation* if (def 4.2.1)
(69) Given linear spaces V and W and a function $T\colon V \to W$, $\mathrm{im}(T) =$ (def 4.2.1)
(70)
(71) Given linear spaces V and W and a function $T\colon V \to W$, $\ker(T) =$ (def 4.2.1)
(72) The *rank* of a linear transformation T is (def 4.2.1)
(73) The *nullity* of a linear transformation T is (def 4.2.1)
(74) A linear transformation T is an isomorphism if (def 4.2.2)
(75) Two vector spaces V and W are isomorphic if (def 4.2.2)
(76) Given an ordered basis $\mathfrak{B}$ of a vector space V, the $\mathfrak{B}$-*matrix* of the linear transformation T is the matrix B satisfying (Def 4.3.1)
(77) Given an ordered bases $\mathfrak{A}$, $\mathfrak{B}$ of a vector space V the *change of basis matrix* from $\mathfrak{B}$ to $\mathfrak{A}$ is the matrix S satisfying (Def 4.3.3)
(78) Two vectors $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^n$ are called *orthogonal* if (Def 5.1.1)
(79) The *length* of a vector $\vec{v} \in \mathbb{R}^n$ is (Def 5.1.1)
(80) A vector $\vec{u}$ in $\mathbb{R}^n$ is a *unit vector* if (Def 5.1.1)
(81) Vectors $\vec{u_1}, \vec{u_2}, \ldots, \vec{u_n} \in \mathbb{R}^n$ are called *orthonormal* if (Def 5.1.2)
(82) The *orthogonal projection* of $\vec{x}$ onto a subspace V is the unique vector such that (Thm 5.1.4)
(83) The *orthogonal complement* of a subspace V of $\mathbb{R}^n$ is the set (def 5.1.7)
(84) The *Cauchy-Schwarz inequality* for vector $\vec{x}$ and $\vec{y} \in \mathbb{R}^n$ is (thm 5.1.11)
(85) The *angle* θ between nonzero vectors $\vec{x}$ and $\vec{y} \in \mathbb{R}^n$ (def 5.1.12)
(86) A linear transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ is called *orthogonal* if (def 5.3.1)
(87) A matrix A is an *orthogonal matrix* if (def 5.3.1)
(88) The *transpose* of an $m \times n$ matrix A is (def 5.3.5)
(89) A square matrix A is *symmetric* if (def 5.3.5)
(90) A square matrix A is *skew-symmetric* if (def 5.3.5)
(91) The *determinant* of A is defined as
(92) Consider an $n \times n$ matrix A. The *minor* A_{ij} is (def 6.2.9)
(93) Consider a linear transformation $T\colon V \to V$, where V is a finite-dimensional linear space. The *determinant* of T is $\det T =$ (def 6.2.11)
(94) The *m-parallelepiped defined by vectors* $\vec{v_1}, \vec{v_2}, \ldots, \vec{v_m}$ in $\mathbb{R}^n$ is (def 6.3.5 p.296)
(95) Consider an $n \times n$ matrix A. The *classical adjoint* $\mathrm{adj}(A)$ is the $n \times n$ matrix with ijth entry (thm 6.3.9)
(96) Consider and $n \times n$ matrix A. A nonzero vector $v \in \mathbb{R}^n$ is called an *eigenvector* of A if (def 7.1.2)
(97) An *eigenvalue* λ of a square matrix A is a scalar such that (def 7.1.2)
(98) The *characteristic equation* of an $n \times n$ matrix A is (Thm 7.2.1)
(99) The *trace* of an $n \times n$ matrix A is (Def 7.2.3)

(100) The *characteristic polynomial* $f_A(\lambda)$ of a square matrix A in variable λ is (thm 7.2.5)
(101) We say that an eigenvalue λ_0 of square matrix A has *algebraic multiplicity* k if (def 7.2.6)
(102) Consider an eigenvalue λ of an $n \times n$ matrix A. The *eigenspace* E_λ associated with λ is (Def 7.3.1)
(103) Consider an eigenvalue λ of an $n \times n$ matrix A. The *geometric multiplicity* of λ is (Def 7.3.2)
(104) Consider an $n \times n$ matrix A. An *eigenbasis* of for A is (Def 7.3.3)
(105) An $n \times n$ matrix A is said to be *diagonalizable* if (Def 7.1.1)