MAS 3105 (Linear Algebra) Your name: KEY

Quiz 7

April 22, 2014

1. A matrix A is called diagonalizable if there are an invertible matrix S and a diagonal matrix D such that $A = SDS^{-1}$

2. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by a matrix

$$A = \begin{bmatrix} -4 & -3 & 0 \\ 6 & 5 & 0 \\ 1 & 4 & 2 \end{bmatrix}$$

(a) Find the characteristic polynomial of A.

$$\chi_A(\lambda) = \det \begin{bmatrix} -4-\lambda & -3 & 0 \\ 6 & 5-\lambda & 0 \\ 1 & 4 & 2-\lambda \end{bmatrix} = (-1)^{3+3} \cdot (2-\lambda) \cdot \det \begin{bmatrix} -4-\lambda & -3 \\ 6 & 5-\lambda \end{bmatrix}$$

$$= (2-\lambda)\left((-4-\lambda)(5-\lambda)+18\right) = (2-\lambda)(\lambda^2-\lambda-2) =$$

$$= (2-\lambda)(\lambda-2)(\lambda+1) = -(\lambda-2)^2(\lambda+1)$$

(b) Find all eigenvalues of T and their algebraic multiplicities.

The eigenvalues are $\lambda_1 = 2$, $\lambda_2 = -1$.
Their corresponding algebraic multiplicities are 2 and 1.