MAS 3105 (Linear Algebra) Your name: KEY

Quiz 6

April 3, 2014

1. A linear transformation $T: \mathbb{R}^n \to \mathbb{R}^n$ is called orthogonal if for each $x \in \mathbb{R}^n$:

$$\|T(\bar{x})\| = \|\bar{x}\|$$

2. (a) Find an orthonormal basis for a subspace V of $\mathbb{R}^3$ spanned by vectors $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$.

We apply Gram-Schmidt process.

$$\bar{u}_1 = \frac{\bar{v}_1}{\|\bar{v}_1\|} = \frac{1}{2}\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\bar{v}_2^{\perp} = \bar{v}_2 - (\bar{v}_2 \cdot \bar{u}_1)\bar{u}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} - \overbrace{\left(\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right)}^{1} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$\bar{u}_2 = \frac{\bar{v}_2^{\perp}}{\|\bar{v}_2^{\perp}\|} = \frac{1}{\sqrt{0^2 + (-1)^2 + 1^2}}\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}.$$

Thus $\mathcal{B} = \left(\bar{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \bar{u}_2 = \begin{bmatrix} 0 \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}\right)$ is an orthonormal basis for V.

(b) Use the results from (a) to find an orthogonal projection of $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ onto V.

$$\mathrm{proj}_V \bar{v}_3 = (\bar{v}_3 \cdot \bar{u}_1)\bar{u}_1 + (\bar{v}_3 \cdot \bar{u}_2)\bar{u}_2 =$$

$$= \underbrace{\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right)}_{=1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \underbrace{\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}\right)}_{=0} \begin{bmatrix} 0 \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$