MAS 3105 (Linear Algebra) Your name: KEY

Quiz 4

February 25, 2014

1. Let $\mathcal{B}$ be a basis of $\mathbb{R}^n$. The $\mathcal{B}$-matrix of a linear transformation $T: \mathbb{R}^n \to \mathbb{R}^n$ is a matrix transforming $[\bar{x}]_{\mathcal{B}}$ into $[T(\bar{x})]_{\mathcal{B}}$.

2. Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 3 \\ -3 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} -4 \\ -1 \\ -3 \end{bmatrix}.$$

Find a linear dependence relation between these vectors (i.e., $c_1, c_2, c_3 \in \mathbb{R}$, not all 0, such that $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 = \mathbf{0}$), thus showing that they are linearly dependent.

$$\begin{bmatrix} 3 & 2 & -4 \\ -3 & -1 & -1 \\ 1 & 1 & -3 \end{bmatrix} \xrightarrow[\text{reduction}]{\text{row}} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{bmatrix}.$$ Therefore, a linear dependence relation is

$$-2\begin{bmatrix} 3 \\ -3 \\ 1 \end{bmatrix} + 5\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} + 1\begin{bmatrix} -4 \\ -1 \\ -3 \end{bmatrix} = \bar{0}$$

3. Consider the following vectors in $\mathbb{R}^2$:

$$\mathbf{v}_1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$$

(a) Find $\mathcal{B}$-coordinate vector $[\mathbf{v}]_{\mathcal{B}}$ of vector $\mathbf{v} = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$.

$$\left[\begin{array}{cc|c} -1 & 2 & -4 \\ 2 & -1 & 5 \end{array}\right] \xrightarrow[\text{reduction}]{\text{row}} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \end{array}\right].$$ Therefore,

$$\begin{bmatrix} -4 \\ 5 \end{bmatrix}_{\mathcal{B}} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

(b) Find a vector $\mathbf{w} \in \mathbb{R}^2$ whose $\mathcal{B}$-coordinate vector is $[\mathbf{w}]_{\mathcal{B}} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

$$\bar{w} = -1\begin{bmatrix} -1 \\ 2 \end{bmatrix} + 1\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$$