Math 3105 (Linear Algebra) Your name: KEY

Quiz 3

February 13, 2014

1. A subset W of $\mathbb{R}^n$ is a subspace if

1) $\bar{0} \in W$
2) $\bar{u}, \bar{v} \in W \Rightarrow \bar{u} + \bar{v} \in W$
3) $c \in \mathbb{R}, \bar{u} \in W \Rightarrow c\bar{u} \in W$

2. Find vectors that span the kernel of a linear transformation $T\colon \mathbb{R}^4 \to \mathbb{R}^2$ defined by a matrix

$$\begin{bmatrix} 1 & -1 & -1 & -1 \\ 2 & -1 & 0 & 1 \end{bmatrix} \xrightarrow{r_2 \to r_2 - 2r_1} \begin{bmatrix} 1 & -1 & -1 & -1 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

$$\xrightarrow{r_1 \to r_1 + r_2} \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 2 & 3 \end{bmatrix}_{\underbrace{\quad\quad}_{\text{basic}} \underbrace{\quad\quad}_{\text{free}}} .$$

Therefore

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -x_3 - 2x_4 \\ -2x_3 - 3x_4 \\ 1 \cdot x_3 + 0\, x_4 \\ 0\, x_3 + 1\, x_4 \end{bmatrix}$$

and

$$\text{span}\left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -3 \\ 0 \\ 1 \end{bmatrix} \right\} = \text{Ker}\, T$$

3. Is the set W of all vectors in $\mathbb{R}^2$ with nonnegative coordinates a subspace of $\mathbb{R}^2$?

No: $\bar{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \in W$, but $(-1) \cdot \bar{w} = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \notin W$,

so condition 3) in the definition of a subspace is violated.