Math 3105 (Linear Algebra) Your name: KEY

Quiz 2

February 6, 2014

1. The kernel of a linear transformation $T: \mathbb{R}^n \to \mathbb{R}^m$ is the set of all vectors in $\mathbb{R}^n$ that are mapped by T to $\bar{0}$.

$$\operatorname{Ker} T = \{\bar{x} \in \mathbb{R}^n : T(\bar{x}) = \bar{0}\}$$

2. Prove that the span of vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ of $\mathbb{R}^m$ is closed under vector addition.

Let $\bar{u}, \bar{v} \in \operatorname{span}\{\bar{v}_1, \ldots, \bar{v}_n\}$. Then

$\bar{u} = c_1\bar{v}_1 + \ldots + c_n\bar{v}_n$ for some $c_i \in \mathbb{R}$

$\bar{v} = b_1\bar{v}_1 + \ldots + b_n\bar{v}_n$ for some $b_i \in \mathbb{R}$

Therefore,

$\bar{u} + \bar{v} = (c_1 + b_1)\bar{v}_1 + \ldots + (c_n + b_n)\bar{v}_n \in \operatorname{span}\{\bar{v}_1, \ldots, \bar{v}_n\}$.

3. What is the image of a linear transformation $S: \mathbb{R}^2 \to \mathbb{R}^3$ defined by the matrix $\begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 3 & -3 \end{bmatrix}$?

$$\operatorname{Im} S = \operatorname{span}\left\{\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}\right\} = \left| \text{as } \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} = (-1)\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} \right| =$$

$$= \operatorname{span}\left\{\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}\right\}.$$

Therefore $\operatorname{Im} S$ is a line in $\mathbb{R}^3$ containing vector $\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$.