

## Quiz 1

January 16, 2014

1. Give a definition of a linear combination of
- $A_1, A_2, \dots, A_n$
- .

A linear combination of  $A_1, \dots, A_n$  is an expression  $c_1 A_1 + c_2 A_2 + \dots + c_n A_n$  for some  $c_1, \dots, c_n \in \mathbb{R}$ .

2. Consider the system of linear equations

$$\begin{cases} x_1 + x_2 - 3x_3 + 2x_4 = 0 \\ 2x_1 + \quad \quad x_3 = -2 \\ \quad \quad x_2 + 4x_3 + 2x_4 = 1 \end{cases}$$

- (a) Reduce the augmented matrix  $A$  of the system to the reduced row echelon form  
 (b) What are the pivot columns in the reduced row echelon form of  $A$ ?  
 (c) Solve the system completely

$$(a) \left[ \begin{array}{cccc|c} 1 & 1 & -3 & 2 & 0 \\ 2 & 0 & 1 & 0 & -2 \\ 0 & 1 & 4 & 2 & 1 \end{array} \right] \xrightarrow{r_2 \rightarrow r_2 - 2r_1} \left[ \begin{array}{cccc|c} 1 & 1 & -3 & 2 & 0 \\ 0 & -2 & 7 & -4 & -2 \\ 0 & 1 & 4 & 2 & 1 \end{array} \right] \xrightarrow{r_2 \leftrightarrow r_3}$$

$$\rightarrow \left[ \begin{array}{cccc|c} 1 & 1 & -3 & 2 & 0 \\ 0 & 1 & 4 & 2 & 1 \\ 0 & -2 & 7 & -4 & -2 \end{array} \right] \xrightarrow{r_3 \rightarrow r_3 + 2r_2} \left[ \begin{array}{cccc|c} 1 & 1 & -3 & 2 & 0 \\ 0 & 1 & 4 & 2 & 1 \\ 0 & 0 & 15 & 0 & 0 \end{array} \right] \xrightarrow{r_3 \rightarrow \frac{r_3}{15}}$$

$$\rightarrow \left[ \begin{array}{cccc|c} 1 & 1 & -3 & 2 & 0 \\ 0 & 1 & 4 & 2 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{array} \right] \xrightarrow{\substack{r_2 \rightarrow r_2 - 4r_3 \\ r_1 \rightarrow r_1 + 3r_3}} \left[ \begin{array}{cccc|c} 1 & 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{array} \right] \xrightarrow{r_1 \rightarrow r_1 - r_2}$$

$$\rightarrow \left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{array} \right] \leftarrow \text{RREF}$$

basic
free

(b) Pivot columns are columns 1, 2, 3.

(c) Express basic variables  $x_1, x_2, x_3$  in terms of a free variable  $x_4$ :

$$\begin{aligned} x_1 &= -1 \\ x_2 &= 1 - 2x_4 \\ x_3 &= 0 \\ x_4 &= \text{anything} \end{aligned}$$