# Suggested Homework Problems
# Topics in Combinatorial and Geometric Group Theory
# Spring 2014

**1.** Prove that subgroups and quotients of countable amenable groups are amenable.

**2.** Is $F_2 \times \mathbb{Z}$ amenable?

**3.** Prove that the following presentations define the trivial group: $\langle a, b \mid ababa, a^2b^3, a^7 \rangle$, $\langle a, b \mid aba^2, b^3, ab^2a \rangle$.

**4.** Prove that the group $\langle a, b \mid aba = bab \rangle$ is an amalgamated free product of two cyclic groups.

**5.** The action of the group $\langle a, b, c \mid a^2, b^2, c^3 \rangle$ on $\{1, 2, 3\}$ is defined by $\pi_a = (1, 2)$, $\pi_b = (2, 3)$, $\pi_c = (1, 2, 3)$. Use the Reidemeister-Schreier algorithm to find the presentation of the stabilizer of point 1.

**6.** Prove that the commutator subgroup of $C_2 * C_3$ is a free group of rank 2.

**7.** Describe the structure of the Cayley graph of the free product of two finite groups.

**8.** Prove that the Cayley graph of an infinite finitely generated group has an infinite two-sided geodesic.

**9.** Prove that the quasi-isometry of metric spaces is an equivalence relation.

**10.** Classify finitely generated abelian groups up to quasi-isometry relation.

**11.** Prove that if a finitely generated group $G$ is quasi-isometric to $\mathbb{Z}$, then it has a finite index subgroup isomorphic to $\mathbb{Z}$.