MAS 3105 (Linear Algebra) Your name: KEY

Quiz 7

November 14, 2013

1. A matrix A is called orthogonal if the columns of A form an orthonormal basis of $\mathbb{R}^n$ (where A is $n \times n$ matrix)

2. (a) Find an orthonormal basis for a subspace V of $\mathbb{R}^3$ spanned by

$$\mathbf{v}_1 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \text{ and } \mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}.$$

$$\bar{u}_1 = \frac{\bar{v}_1}{\|\bar{v}_1\|} = \frac{1}{2}\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\bar{v}_2^{\perp} = \bar{v}_2 - (\bar{v}_2 \cdot \bar{u}_1) \cdot \bar{u}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} - \left(\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right) \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$\bar{u}_2 = \frac{\bar{v}_2^{\perp}}{\|\bar{v}_2^{\perp}\|} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix}.$$

So an orthonormal basis for V is

$$\left(\bar{u}_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \bar{u}_2 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix} \right)$$

(b) Use result from (a) to find an orthogonal projection of $\mathbf{v}_3 = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ onto V.

$$\text{Proj}_V \bar{v}_3 = (\bar{v}_3 \cdot \bar{u}_1)\bar{u}_1 + (\bar{v}_3 \cdot \bar{u}_2)\bar{u}_2 = \left(\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right) \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \left(\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix} \right) \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix} =$$

$$= \frac{2}{\sqrt{2}} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$