MAS 3105 (Linear Algebra) Your name: KEY

Quiz 5

October 17, 2013

1. Let $\mathcal{B}$ be a basis of $\mathbb{R}^n$. The $\mathcal{B}$-matrix of a linear transformation $T: \mathbb{R}^n \to \mathbb{R}^n$ is an $n \times n$ matrix B s.t.

$$[T(\bar{x})]_{\mathcal{B}} = B \cdot [\bar{x}]_{\mathcal{B}}$$

for each $\bar{x} \in \mathbb{R}^n$.

2. Consider the following vectors in $\mathbb{R}^2$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ -3 \end{bmatrix}.$$

(a) Prove that $\mathcal{B} = (\mathbf{v}_1, \mathbf{v}_2)$ is a basis for $\mathbb{R}^2$.

Since $\bar{v}_1$ and $\bar{v}_2$ are not collinear, they are linearly independent. Any 2 linearly independent vectors of $\mathbb{R}^2$ form a basis for $\mathbb{R}^2$

(b) Find $\mathcal{B}$-coordinate vector $[\mathbf{v}]_{\mathcal{B}}$ of vector $\mathbf{v} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$.

$$\begin{array}{ccc} \bar{v}_1 & \bar{v}_2 & \bar{v} \end{array}$$

$$\left[\begin{array}{cc|c} 1 & 2 & 0 \\ -2 & -3 & 2 \end{array}\right] \xrightarrow{r_2 \to r_2 + 2r_1} \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 1 & 2 \end{array}\right] \xrightarrow{r_1 \to r_1 - 2r_2} \left[\begin{array}{cc|c} 1 & 0 & -4 \\ 0 & 1 & 2 \end{array}\right]$$

So $[\bar{v}]_{\mathcal{B}} = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$

(b) Find a vector $\mathbf{w} \in \mathbb{R}^2$ whose $\mathcal{B}$-coordinate vector is $[\mathbf{w}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

$$\bar{w} = \begin{array}{cc} \bar{v}_1 & \bar{v}_2 \\ \end{array} \begin{bmatrix} 1 & 2 \\ -2 & -3 \end{bmatrix} \cdot \overset{[w]_{\mathcal{B}}}{\begin{bmatrix} 2 \\ 1 \end{bmatrix}} = \begin{bmatrix} 1 \cdot 2 + 2 \cdot 1 \\ -2 \cdot 2 + (-3) \cdot 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -7 \end{bmatrix}$$