MAS 3105 (Linear Algebra) Your name: KEY

Quiz 4

October 10, 2013

1. A collection of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_m\}$ in a subspace V of $\mathbb{R}^n$ is called a basis of V if

1) $\operatorname{span}\{\bar{v}_1, \ldots, \bar{v}_m\} = V$

and

2) $\{\bar{v}_1, \ldots, \bar{v}_m\}$ is linearly independent

2. Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} -4 \\ -1 \\ -3 \end{bmatrix}.$$

(a) Prove that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent.

Consider the matrix A with columns $\bar{v}_1, \bar{v}_2, \bar{v}_3$:

$$A = \begin{bmatrix} 1 & 2 & -4 \\ -2 & -1 & -1 \\ 0 & 1 & -3 \end{bmatrix} \xrightarrow{r_2 \to r_2 + 2r_1} \begin{bmatrix} 1 & 2 & -4 \\ 0 & 3 & -9 \\ 0 & 1 & -3 \end{bmatrix} \xrightarrow{r_3 \to r_3 - \frac{1}{3} r_2} \begin{bmatrix} 1 & 2 & -4 \\ 0 & 3 & -9 \\ 0 & 0 & 0 \end{bmatrix} \text{REF}$$

Since the rank of $A = 2 < 3 = \#$ of columns of A, vectors $\bar{v}_1, \bar{v}_2, \bar{v}_3$ are linearly dependent.

(b) Find a linear dependence relation between these vectors (i.e., $c_1, c_2, c_3 \in \mathbb{R}$, not all 0, such that $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 = \mathbf{0}$).

To find c_1, c_2, c_3 we reduce A to RREF:

$$\begin{bmatrix} 1 & 2 & -4 \\ 0 & 3 & -9 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{r_2 \to \frac{r_2}{3}} \begin{bmatrix} 1 & 2 & -4 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{r_1 \to r_1 - 2r_2} \begin{array}{c} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{matrix} c_1 & c_2 & c_3 \end{matrix} \end{array}$$

Therefore,

$$\begin{aligned} c_1 &= -2c_3, \\ c_2 &= 3c_3, \\ c_3 &= c_3, \end{aligned}$$

so setting $c_3 = 1$ yields

$\boxed{c_1 = -2,\ c_2 = 3,\ c_3 = 1}$ and a linear dependence relation

is $-2\bar{v}_1 + 3\bar{v}_2 + 1\bar{v}_3 = \bar{0}$.