MAS 3105 (Linear Algebra) Your name: KEY

Quiz 3

October 3, 2013

1. Give the definition of a span of the vectors $v_1, v_2, \ldots, v_m$ in $\mathbb{R}^n$.

$\mathrm{span}\{\bar{v}_1, \ldots, \bar{v}_m\} = \{c_1\bar{v}_1 + \ldots + c_m\bar{v}_m : c_i \in \mathbb{R}\}$

is a collection of all linear combinations of $\bar{v}_1, \ldots, \bar{v}_m$

2. Let $T: \mathbb{R}^4 \to \mathbb{R}^2$ be a linear transformation defined by a matrix

$$A = \begin{bmatrix} 1 & 2 & 1 & -1 \\ -2 & -4 & -1 & 3 \end{bmatrix}$$

(a) Find vectors that span the image of T.

$$\mathrm{Im}(T) = \mathrm{Span}\left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \end{bmatrix} \right\}$$

(b) Find vectors that span the kernel of T.

$$\begin{bmatrix} 1 & 2 & 1 & -1 \\ -2 & -4 & -1 & 3 \end{bmatrix} \xrightarrow{r_2 \to r_2 + 2r_1} \begin{bmatrix} 1 & 2 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{r_1 \to r_1 - r_2}$$

$$\to \begin{bmatrix} 1 & 2 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix}. \quad \text{So}$$

$$\mathrm{Ker}\,T = \left\{ \begin{bmatrix} -2x_2 + 2x_4 \\ 1x_2 + 0x_4 \\ 0x_2 - 1\cdot x_4 \\ 0x_2 + 1x_4 \end{bmatrix} : x_2, x_4 \in \mathbb{R} \right\} = \mathrm{span}\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\}$$