MAS 3105 (Linear Algebra) Your name: KEY

Quiz 2

September 12, 2013

1. Give a definition of a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$.

A linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ is a map $T: \mathbb{R}^n \to \mathbb{R}^m$ such that for all $\bar{u}, \bar{v} \in \mathbb{R}^n$ and all $k \in \mathbb{R}$

$$\left.\begin{aligned} &1)\ T(\bar{u}+\bar{v}) = T(\bar{u}) + T(\bar{v}) \\ &2)\ T(k\bar{u}) = kT(\bar{u}) \end{aligned}\right\}$$

2. Let

$$A = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 0 & 1 \\ 3 & 0 & 0 \end{bmatrix}$$

State which of the following is defined and compute corresponding expressions.

(a) BA

$$\begin{bmatrix} 2 & 0 & 1 \\ -1 & 0 & 1 \\ 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2\cdot 0 + 0(-1) + 1\cdot 2 \\ -1\cdot 0 + 0\cdot(-1) + 1\cdot 2 \\ 3\cdot 0 + 0\cdot(-1) + 0 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

(b) A+B

Not Defined

(c) 2A

$$2\begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 4 \end{bmatrix}$$

3. Let T be a linear transformation defined by a matrix pivot columns

$$C = \begin{bmatrix} 1 & 2 & 1 & -1 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

(a) What is the domain of T? $= \mathbb{R}^{\#\text{ of columns}} = \mathbb{R}^4$

(b) What is the codomain of T? $= \mathbb{R}^{\#\text{ of rows}} = \mathbb{R}^3$

(c) Is T one-to-one? Explain.

No, because $\operatorname{rank} C = 3 \neq 4 = \#$ of columns

(d) Is T onto? Explain.

Yes, because $\operatorname{rank} C = 3 = \#$ of rows