

Quiz 1

September 5, 2013

1. Give a definition of a linear combination of A_1, A_2, \dots, A_n .

A linear combination of A_1, A_2, \dots, A_n is an expression of the form

$$c_1 A_1 + c_2 A_2 + \dots + c_n A_n, \text{ where } c_i \text{ 's are real numbers}$$

2. Determine if the following system of linear equations is consistent

$$\begin{cases} x_1 - 2x_2 + x_3 - 4x_4 = 1 \\ x_1 + 3x_2 + 7x_3 + 2x_4 = 2 \\ x_1 - 12x_2 - 11x_3 - 16x_4 = -1 \end{cases}$$

The augmented matrix of a system is

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -4 & 1 \\ 1 & 3 & 7 & 2 & 2 \\ 1 & -12 & -11 & -16 & -1 \end{array} \right] \xrightarrow{\substack{r_3 \rightarrow r_3 - r_1 \\ r_2 \rightarrow r_2 - r_1}} \left[\begin{array}{cccc|c} 1 & -2 & 1 & -4 & 1 \\ 0 & 5 & 6 & 6 & 1 \\ 0 & -10 & -12 & -12 & -2 \end{array} \right] \xrightarrow{r_3 \rightarrow r_3 + 2r_2} \left[\begin{array}{cccc|c} 1 & -2 & 1 & -4 & 1 \\ 0 & 5 & 6 & 6 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Since the last column of REF does not contain a leading entry, the system is consistent.

3. Consider the system of linear equations

$$\begin{cases} 5x_1 - 2x_2 + 6x_3 = 0 \\ -2x_1 + x_2 + 3x_3 = 1 \end{cases}$$

- (a) Reduce the augmented matrix A of the system to the reduced row echelon form
- (b) What are the pivot columns in the reduced row echelon form of A ?
- (c) What is the rank of A ?
- (d) Solve the system completely

(a)
$$\left[\begin{array}{ccc|c} 5 & -2 & 6 & 0 \\ -2 & 1 & 3 & 1 \end{array} \right] \xrightarrow{r_1 \rightarrow r_1 + 2r_2} \left[\begin{array}{ccc|c} 1 & 0 & 12 & 2 \\ -2 & 1 & 3 & 1 \end{array} \right] \xrightarrow{r_2 \rightarrow r_2 + 2r_1} \left[\begin{array}{ccc|c} 1 & 0 & 12 & 2 \\ 0 & 1 & 27 & 5 \end{array} \right] \leftarrow \text{RREF}$$

- (b) Pivot columns are columns 1 and 2.
- (c) rank $A = \#$ of pivot columns = 2

(d)
$$\boxed{\begin{matrix} x_1 = 2 - 12x_3 \\ x_2 = 5 - 27x_3 \\ x_3 = x_3 \end{matrix}}$$
 — general solution.