Math 375 (Complex Analysis) Your name: KEY

Quiz 8

April 27, 2012

1. A sequence of functions $\{f_n\}_{n\geq 0}$ from set $G \subset \mathbb{C}$ to $\mathbb{C}$ converges uniformly to a function $f\colon G \to \mathbb{C}$ if $\forall \varepsilon > 0 \ \exists N \geq 0 \ \forall n \geq N \ \forall z \in G : |f_n(z) - f(z)| < \varepsilon$

2. Determine where the series $\sum_{k\geq 0} \frac{2^k}{k!} z^{2k}$ converges and find the function represented by the series. We have

$$\sum_{k\geq 0} \frac{2^k}{k!} z^{2k} = \sum_{k\geq 0} \frac{(2z^2)^k}{k!} = e^{2z^2}.$$

Since e^{2z^2} is an entire function, the radius of convergence of the series is ∞.

3. Compute the power series expansion of $f(z) = \frac{z^2}{z+2i}$ centered at $z_0 = 0$ and find the radius of convergence of the obtained series.

$$\frac{z^2}{z+2i} = \frac{1}{2i} \frac{z^2}{1-\left(-\frac{z}{2i}\right)} = \frac{1}{2i} \cdot \frac{z^2}{1 - \frac{iz}{2}} = \frac{1}{2i} \cdot z^2 \cdot \sum_{k\geq 0} \left(\frac{iz}{2}\right)^k =$$

$$= \sum_{k\geq 0} \frac{i^{k-1}}{2^{k+1}} \cdot z^{k+2}.$$

Since $\frac{z^2}{z+2i}$ is holomorphic in $D_2(0)$ and not holomorphic in $D_r(0)$ for $r > 2$ we get that the radius of conv. of the series is 2.