Math 375 (Complex Analysis) Your name: KEY

Quiz 5

March 15, 2012

1. Let G be a region in $\mathbb{C}$. Two simple closed curves γ_0 and γ_1 in G are called G-homotopic if

there is a continuous function $h: [0,1]^2 \to G$

s.t.
$$h(t,0) = \gamma_0(t)$$
$$h(t,1) = \gamma_1(t) \quad \forall s,t \in [0,1]$$
$$h(0,s) = h(1,s)$$

where $\gamma_0(t)$, $\gamma_1(t)$ are parametrizations of γ_0 and γ_1

2. Compute $\int_\gamma \bar{z}\, dz$ where γ is a semicircle from 1 through i to -1.

We have $\gamma(t) = e^{it}$, $t \in [0, \pi]$, so

$$\int_\gamma \bar{z}\, dz = \int_0^\pi e^{-it} \cdot i e^{it}\, dt = \int_0^\pi i\, dt = \pi i.$$

3. Find the length of the curve γ parameterized by $\gamma(t) = \frac{1}{2}t^2 + i\frac{1}{3}t^3$, $t \in [0,2]$.

$$\text{length}(\gamma) = \int_0^2 |\gamma'(t)|\, dt = \int_0^2 |t + it^2|\, dt = \int_0^2 \sqrt{t^2 + t^4}\, dt =$$

$$= \int_0^2 t\sqrt{1+t^2}\, dt = \left| \begin{matrix} u = t^2+1 \\ du = 2t\, dt \end{matrix} \quad \begin{array}{c|c} u & t \\ \hline 1 & 0 \\ 5 & 2 \end{array} \right| = \frac{1}{2}\int_1^5 \sqrt{u}\, du = \frac{1}{2} \cdot \frac{2}{3} u^{\frac{3}{2}} \Big|_1^5 =$$

$$= \frac{1}{3}\left(5\sqrt{5} - 1\right)$$