Math 375 (Complex Analysis) Your name: Key

Quiz 4

March 8, 2012

1. Give a definition of a branch of a complex logarithm.

A continuous function $\mathrm{Log}\, z$ defined on a region $U \subseteq \mathbb{C}$ satisfying $\exp(\mathrm{Log}\, z) = z \quad \forall z \in U$ is called a branch of a logarithm

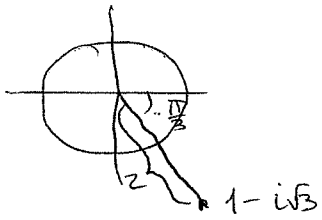
2. (a) Find all solutions of the equation $e^z = 1 - i\sqrt{3}$.

Let $z = x + iy$.

$e^z = e^x \cdot e^{iy} = 1 - i\sqrt{3} = 2e^{i(-\frac{\pi}{3})}$

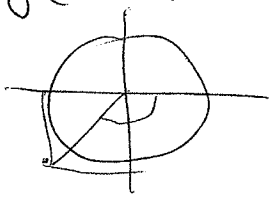
So $x = \ln 2, \quad y = -\frac{\pi}{3} + 2\pi k$

$z = \ln 2 + i\left(-\frac{\pi}{3} + 2\pi k\right), \quad k \in \mathbb{Z}$



(b) Find the principal value of $\log(-1 - i)$.

$\mathrm{Log}(-1-i) = \ln|-1-i| + i\,\mathrm{Arg}(-1-i) = \ln\sqrt{2} + i\cdot\left(-\frac{3\pi}{4}\right)$



3. Evaluate principal values of the following expressions in the form $x + iy$.

(a) $2^i = \exp(i\,\mathrm{Log}\, 2) = e^{i\ln 2} = \cos(\ln 2) + i\sin(\ln 2)$

(b) $i^i = \exp(i\,\mathrm{Log}\, i) = \exp\left(i\cdot\left(0 + i\cdot\frac{\pi}{2}\right)\right) = e^{-\frac{\pi}{2}} + 0\cdot i$