Math 375 (Complex Analysis) Your name: KEY

Quiz 3

February 29, 2012

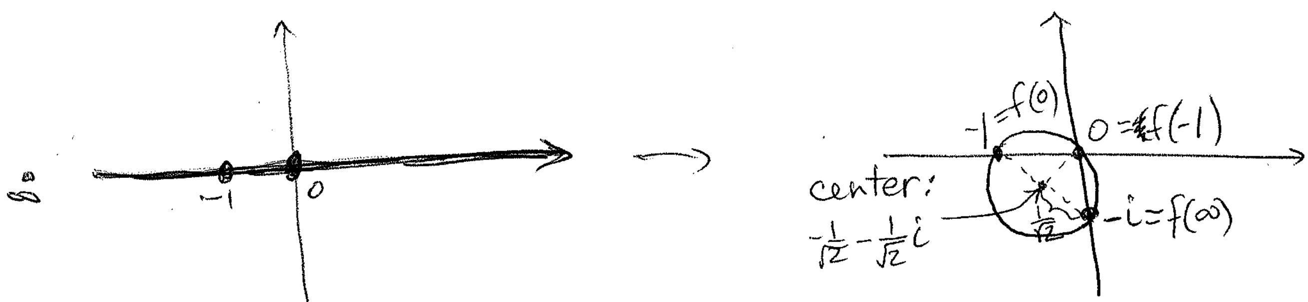
1. Give a definition of a Möbius transformation.

A Möbius transformation is a map $f : \mathbb{C} \to \mathbb{C}$ defined by $f(z) = \frac{az+b}{cz+d}$ for some $a, b, c, d \in \mathbb{C}$ s.t. $ad - bc \neq 0$.

2. Let $f(z) = \frac{z+1}{iz-1}$. Find the image of the real axis under f.

3 points on x-axis are $0, -1, \infty$.

$f(0) = \frac{1}{-1} = -1$, $f(-1) = 0$, $f(\infty) = \frac{1}{i} = -i$



So $f(\{\mathrm{Im}\, z = 0\}) = \left\{ z : \left| z - \left(-\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right) \right| = \frac{1}{\sqrt{2}} \right\}$

3. Find the Möbius transformation f that map $0 \to i$, $i \to \infty$ and $-1 \to 0$.

Let $f(z) = \frac{az+b}{cz+d}$

$f(0) = i \Rightarrow \frac{b}{d} = i \Rightarrow b = id$

$f(i) = \infty \Rightarrow \frac{ai+b}{ci+d} = \infty \Rightarrow ci + d = 0 \Rightarrow d = -ic$

$f(-1) = 0 \Rightarrow \frac{-a+b}{-c+d} = 0 \Rightarrow a = b$

Set $c = 1$. Then $d = -ic = -i$, $b = id = i(-i) = 1$, $a = b = 1$.

So $\boxed{f(z) = \frac{z+1}{z-i}}$