Math 375 (Complex Analysis) Your name: KEY

Quiz 2

February 17, 2012

1. A function $f\colon \mathbb{C} \to \mathbb{C}$ is called holomorphic at $z_0 \in \mathbb{C}$ if there exists $r>0$ s.t. f is differentiable at each point of $D_r(z_0)$

2. Evaluate the following limits or explain why they don't exist (here $z = x + iy$).

(a) $\lim_{z\to -i} \frac{z^3 - i}{z+i} = \lim_{z\to -i} \frac{z^3 + i^3}{z+i} = \lim_{z\to -i} \frac{(z+i)(z^2 - iz + i^2)}{z+i} =$

$= \lim_{z\to -i} (z^2 - iz - 1) = (-i)^2 - i\cdot(-i) - 1 = -1 - 1 - 1 = -3$

(b) $\lim_{z\to 1+2i} x + i(x^2 - y) \quad = \quad 1 + i(1^2 - 2) = 1 - i$

3. Is the function $f(z) = \mathrm{Re}(z)$ differentiable at some point in $\mathbb{C}$? Why?

$f(z) = f(x,y) = x + 0\cdot i = u(x,y) + iv(x,y)$.

So we have $u(x,y) = x$, $v(x,y) = 0$.

We see that $u_x = 1 \neq 0 = v_y$, so Cauchy-Riemann equations are not satisfied and f cannot be differentiable at any point.