Math 375 (Complex Analysis) Your name: KEY

Quiz 1

February 8, 2012

1. Give a definition of an open set in $\mathbb{C}$.

A set $D \subset \mathbb{C}$ is called open if $\text{int}(D) = D$

or

A set $D \subset \mathbb{C}$ is called open if for each $x \in D$ there is $r > 0$ s.t. $D_r(x) \subset D$.

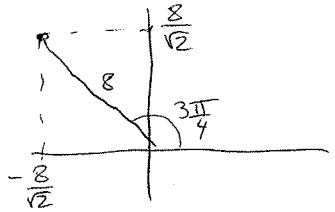
2. Find real and imaginary parts of the following complex numbers:

(a) $\frac{5+10i}{3-4i} = \frac{5+10i}{(3-4i)} \cdot \frac{(3+4i)}{(3+4i)} = \frac{5 \cdot (1+2i)(3+4i)}{3^2+4^2} =$

$= \frac{1}{5}\left((1\cdot 3 - 2\cdot 4) + (2\cdot 3 + 1\cdot 4)i\right) = -1 + 2i$

So $\text{Re } z = -1$, $\text{Im } z = 2$

(b) $(2e^{i\frac{\pi}{4}})^3 = 2^3 \cdot e^{i\frac{3\pi}{4}} = \left(-\frac{8}{\sqrt{2}}\right) + i\left(\frac{8}{\sqrt{2}}\right)$

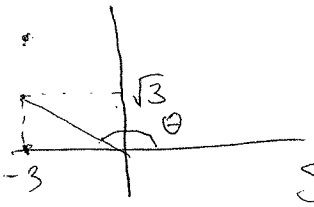


So $\text{Re } z = \frac{-8}{\sqrt{2}}$, $\text{Im } z = \frac{8}{\sqrt{2}}$

3. Write in polar form:

(a) $-3 + \sqrt{3}i$

$\theta = \arctan\frac{\sqrt{3}}{(-3)} + \pi = -\frac{\pi}{6} + \pi = \frac{5\pi}{6}$



$r = \sqrt{(-3)^2 + (\sqrt{3})^2} = \sqrt{12} = 2\sqrt{3}$

So $z = 2\sqrt{3} \cdot e^{i \cdot \frac{5\pi}{6}}$

(b) $(1-i)^3 = \left(\sqrt{2} \cdot e^{(-\frac{\pi}{4})\cdot i}\right)^3 = \sqrt{8} \cdot e^{(-\frac{3\pi}{4})i}$