1 Math 375 Final Practice Questions

1. Find all solutions to $z^6 = -9$.
2. Show that if f and $\overline{f}$ are both holomorphic in a region G, then f is constant in G.
3. State the Cauchy-Riemann equations.
4. Find the Möbius transformations satisfying each of the following. Write your answers in standard form:
   (a) $0 \mapsto 0$, $1 \mapsto 1$, $\infty \mapsto 2$
   (b) $i \mapsto -1$, $2i \mapsto -2$, $0 \mapsto 0$.
5. Find the fixed points in $\mathbb{C}$ of $f(z) = \frac{z^2}{2z+i}$.
6. Find all solutions to the equation $\exp(z) = \pi i$.
7. Find the length of the curve γ parametrized by $\gamma(t) = e^{it} + i\pi e^{it}$ for $0 \leq t \leq \pi$.
8. Compute $\int_\gamma z^2 dz$, where γ is the semicircle from 1 through i to -1.
9. Prove Liouville's Theorem: Every bounded entire function is constant.
10. State the First Fundamental Theorem of Calculus.
11. Show that $e^x \sin y$ is harmonic on $\mathbb{C}$.
12. Prove that $\mathbb{Z}$ is complete.
13. Derive a formula for the product of two power series.
14. State the Maximum-Modulus Theorem.
15. Find the Taylor series about 0 for the following functions.
    (a) $(z^2 - 1)e^z$
    (b) $\frac{1}{1+z}$
    (c) $\frac{1}{e^z}$
16. Find the multiplicities of all zeros of $(1 + z^2)^3$.
17. Prove that if f is entire and constant on the disk $D_1(0)$ then f is constant.
18. Let γ be the circle of radius 3 centered at 0. Compute the following integrals.
    (a) $\int_\gamma \frac{1}{z^2+1} dz$
    (b) $\int_\gamma \frac{\sin z}{z^2} dz$
    (c) $\frac{\tan(z)\exp(z)}{(z-1)^2}$
19. Find the poles of $\frac{z}{1-e^z}$ and determine their orders.