Math 461 (Topology I) Your name: KEY

Quiz 4

October 28, 2011

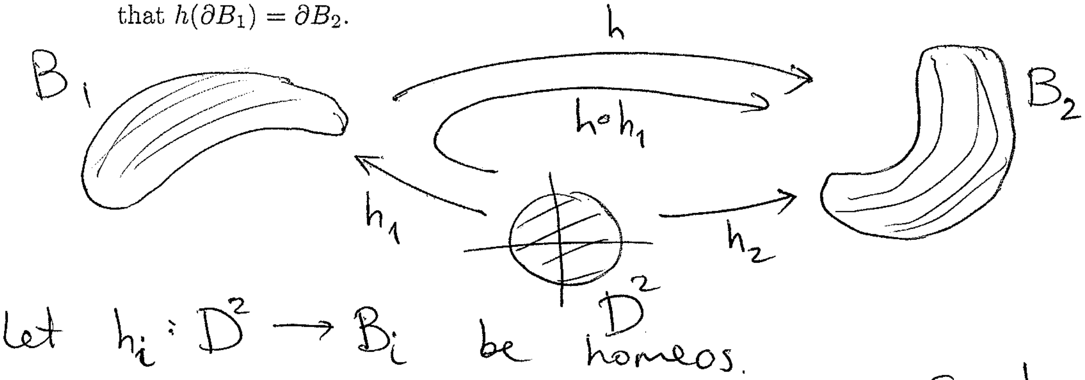
1. State the invariance of a domain theorem.

If $A \subset \mathbb{R}^n$ and $A \cong \mathbb{R}^n$, then A must be open in $\mathbb{R}^n$

2. Show that a surface from which a closed subset has been removed is still a surface.

Let S be a surface and $K \subset S$ be a closed subset.
Let $x \in S-K$ be arbitrary. S -surface $\Rightarrow$ $\exists\, U \subset S$ - nbhd of x s.t. $U \cong \mathbb{R}^2$. Suppose $h: U \to \mathbb{R}^2$ is a homeo. Then $U \cap (S-K)$ is open in U and $x \in U \cap (S-K)$. So $h(x) \in \underbrace{h(U \cap (S-K))}_{\text{open}}$
$\Rightarrow \exists\, B_\varepsilon(h(x)) \subset h(U \cap (S-K))$. Then
$h^{-1}(B_\varepsilon(h(x))) \subset U \cap (S-K) \subset S-K$ and we have $x \in h^{-1}(B_\varepsilon(h(x)))$ and $h^{-1}(B_\varepsilon(h(x))) \cong B_\varepsilon(h(x)) \cong \mathbb{R}^2$.

3. Let $B_1 \subset \mathbb{R}^m$ and $B_2 \subset \mathbb{R}^n$ be two disks and let $h\colon B_1 \to B_2$ be a homeomorphism. Prove that $h(\partial B_1) = \partial B_2$.



Let $h_i: D^2 \to B_i$ be homeos.
∂B_2 does not depend on a choice of h_2. So
$\partial B_2 = h(\underbrace{h_1(\partial D^2)}_{\partial B_1}) = h(\partial B_1)$.