Math 461 (Topology I) Your name: KEY

Quiz 3

October 12, 2011

1. Give a definition of a compact topological space.

A topological space is called compact if every open cover of X contains a finite subcover

2. Prove that a union of finite number of compact subspaces of a topological space is compact.

Suppose $A_1, \dots, A_n$ are compact subspaces of X. Let $\mathcal{U} = \{U_\alpha, \alpha \in I\}$ be an open cover of $\bigcup_{i=1}^{n} A_i$. Then $\mathcal{U}$ is also an open cover for each of A_i's. A_i's are compact, so $\forall i$ $\exists$ a finite subcover $\{U_{\alpha_1}^{(i)}, \dots, U_{\alpha_{k_i}}^{(i)}\}$ of $\mathcal{U}$ that covers A_i. The union of these finite subcovers is a finite subcover of $\mathcal{U}$ that covers $\bigcup_{i=1}^{n} A_i$

3. Is $[0, 1]$ homeomorphic to a circle? Explain!

No: each point except 0 and 1 in $[0,1]$ is a cut point, while there are no cut points in the circle.