Math 372 (Dynamical systems) Fall 2010

# Review problems for the midterm

**Note:** I will not collect this assignment – just do it for your benefit. This is a preparational homework for the final that covers the topics that will be presented on the midterm. The midterm will be over Chapters 2-6 in the text.

## Solve the following problems

1. For each of the following linear systems determine:
   - find general solution
   - determine the stability type of the origin
   - determine the directions of expansion and contraction (corresponding the eigenvectors)
   - sketch the phase portrait

   (a) $\dot{\mathbf{x}} = \begin{bmatrix} 2 & 9 \\ 0 & -1 \end{bmatrix} \cdot \mathbf{x}$

   (b) $\dot{\mathbf{x}} = \begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix} \cdot \mathbf{x}$

   (c) $\dot{\mathbf{x}} = \begin{bmatrix} 10 & -18 \\ 5 & -9 \end{bmatrix} \cdot \mathbf{x}$

   (d) $\dot{\mathbf{x}} = \begin{bmatrix} -2 & 1 \\ -16 & 6 \end{bmatrix} \cdot \mathbf{x}$

   (e) $\dot{\mathbf{x}} = \begin{bmatrix} -1 & -2 & 1 \\ 0 & 2 & 0 \\ 0 & 6 & -1 \end{bmatrix} \cdot \mathbf{x}$

   (f) $\dot{\mathbf{x}} = \begin{bmatrix} -1 & -6 & 3 \\ 0 & -3 & 0 \\ 0 & -2 & -2 \end{bmatrix} \cdot \mathbf{x}$

2. For each of the following differential equations
   - find all fixed points
   - plot a phase portrait
   - solve for an explicit solution
   - what is the maximal interval where the flow $\phi(t, \mathbf{x}_0)$ is defined

   (a) $\dot{x} = x^2 + 2x - 3$

   (b) $\dot{x} = x^n$ for $n \geq 1$

   (c) $\dot{x} = x^2 + 4$.

3. For the following systems of differential equations
   - find all fixed points
   - determine the nullclines and signs of $\dot{x}$ and $\dot{y}$ in the various regions of the plane
   - for each fixed point determine the linearized system at this point, compute the type of the linearized system and check if you can deduce something about the stability type of this fixed point

- for saddle fixed points find the eigenvectors that correspond to the expanding and attracting directions near these points
- sketch a phase portrait (with some trajectories)

(a) $\begin{cases} \dot{x} = y - x^3, \\ \dot{y} = -x - y. \end{cases}$

(b) $\begin{cases} \dot{x} = -\frac{x}{2} + y, \\ \dot{y} = 1 - y^2. \end{cases}$

(c) $\begin{cases} \dot{x} = (x-1)(y-1), \\ \dot{y} = 3 - xy. \end{cases}$

4. Do problems 5.2.3 (after you do the homework with 5.2.1, 5.2.2 and 5.2.4), 5.3.2.

5. Do problems 6.1.1, 6.2.1, 6.2.2 (perhaps, I will not give such problem on a midterm, but it may be on a final or in the next homework).