

Quiz 8

November 12, 2010

1. Let T be the tent map. Recall that T^n has 2^n fixed points. How many orbits of size 4 does T have? Explain.

T^4 has $2^4 = 16$ fixed points, 4 of which are fixed by T^2 (those are fixed and period 2 points). Therefore there are $16 - 4 = 12$ period-4 points and $3 = 12/4$ period-4 orbits.

Answer: $\boxed{3}$

2. Consider a discrete dynamical system given by a function $F: \mathbb{R} \rightarrow \mathbb{R}$, $F(x) = x^2 - 2x + 2$.

- Determine the fixed points

$$F(x) = x$$

$$x^2 - 2x + 2 = x$$

$$x^2 - 3x + 2 = 0 \Rightarrow x = 1, \quad x = 2$$

- Classify fixed points as attracting or repelling by looking at the derivative of $F(x)$.

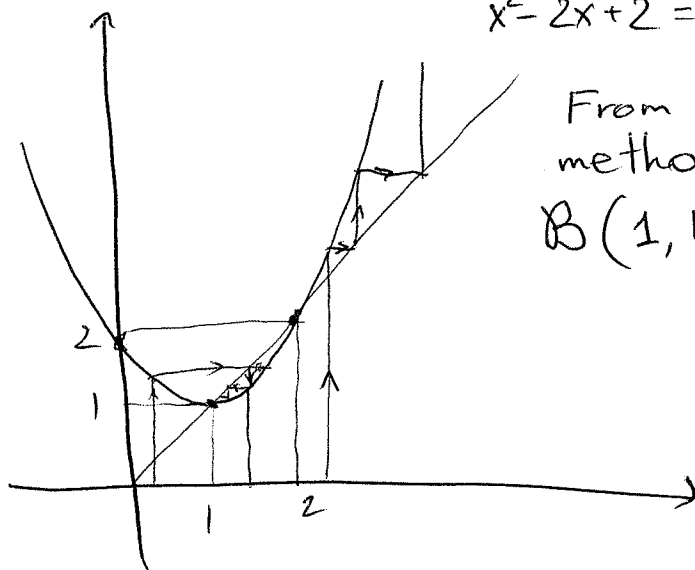
$$F'(x) = 2x - 2$$

$$|F'(1)| = |2 - 2| = 0 < 1 \Rightarrow 1 \text{ is a superattracting point}$$

$$|F'(2)| = |4 - 2| = 2 > 1 \Rightarrow 2 \text{ is a repelling point}$$

- Verify your answer in previous part using the graphical iterations method.

$$x^2 - 2x + 2 = (x-1)^2 + 1$$



From graphical iteration method it follows that $B(1, F) = (0, 2)$.