

## Quiz 7

November 2, 2010

1. Give a definition of a periodic point of period  $k$  for a dynamical system given by function  $F(x)$ .

A point  $x^*$  is called  $k$ -periodic for  $F$  if  $F^k(x^*) = x^*$  and  $F^l(x^*) \neq x^*$  for all  $0 < l < k$ .

2. Consider a discrete dynamical system given by a function  $F: \mathbb{R} \rightarrow \mathbb{R}$ ,  $F(x) = x^2 - 2$ .

- Determine the fixed points

$$F(x) = x \Rightarrow x^2 - 2 = x \Rightarrow x^2 - x - 2 = 0 \Rightarrow \begin{matrix} x = -1 \\ x = 2 \end{matrix}$$

- Calculate the explicit expression for  $F^2(x)$

$$F^2(x) = F(x^2 - 2) = (x^2 - 2)^2 - 2 = x^4 - 4x^2 + 2$$

- Find points of period 2 for  $F$  (hint: you might want to use answer from the first item)

Need to solve  $F^2(x) = x$ .

$$x^4 - 4x^2 + 2 = x \Rightarrow x^4 - 4x^2 - x + 2 = 0$$

As  $x = -1, x = 2$  are fixed points for  $F$  we can factor  $(x+1)(x-2)$  in  $F^2(x) - x$ .

$$F^2(x) - x = x^4 - 4x^2 - x + 2 = x^2(x^2 - 4) - (x - 2) =$$

$$= x^2(x+2)(x-2) - (x-2) = (x-2)(x^3 + 2x^2 - 1) =$$

$$= (x^3 + x^2 + x^2 - 1)(x-2) = (x^2(x+1) + (x-1)(x+1))(x-2) =$$

$$= (x-2)(x+1)(x^2 + x - 1) = 0$$

$$D = 1 + 4 = 5 \Rightarrow \boxed{x = \frac{-1 + \sqrt{5}}{2}, x = \frac{-1 - \sqrt{5}}{2}}$$

are period 2 points for  $F$ .

Alternatively,  
one can do  
long division