

## Quiz 5

October 13, 2010

1. Consider a particle moving whose position  $x$  and velocity  $y$  are governed by the following conservative system of differential equations.

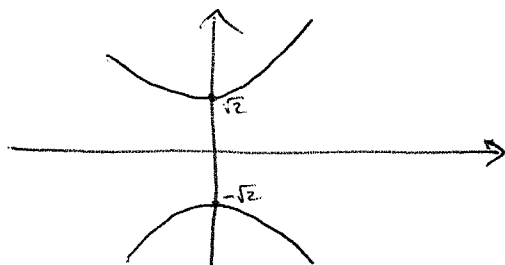
$$\begin{aligned}\dot{x} &= y, \\ \dot{y} &= x.\end{aligned}$$

- Compute the total energy function  $E(x, y)$  preserved along the solutions of the above system.

$$E(x, y) = \frac{y^2}{2} + V(x) = \frac{y^2}{2} + \int_0^x (-s) ds = \frac{y^2}{2} - \frac{x^2}{2}$$

- Sketch the level set  $E^{-1}(1)$ .

$$E^{-1}(1) = \{(x, y) : \frac{y^2}{2} - \frac{x^2}{2} = 1\} \text{ - hyperbola}$$



2. Consider a nonlinear system of differential equations

$$\begin{aligned}\dot{x} &= y - x^2, \\ \dot{y} &= x + y.\end{aligned}$$

- Determine the fixed points

$$\begin{cases} y - x^2 = 0 \\ x + y = 0 \end{cases} \Rightarrow \begin{cases} y = -x \\ -x - x^2 = 0 \end{cases} \Rightarrow \begin{cases} x(x+1) = 0 \\ y = -x \end{cases} \Rightarrow \begin{matrix} (0, 0) \\ (-1, 1) \end{matrix} \text{ are fixed points}$$

- Determine the nullclines and signs of  $\dot{x}$  and  $\dot{y}$  in various regions of the plane

Nullclines are

$$\begin{aligned}\dot{x} = y - x^2 = 0 &\Rightarrow y = x^2 \\ \dot{y} = x + y = 0 &\Rightarrow y = -x\end{aligned}$$

- Sketch a phase portrait.

