Math 372 (Dynamical Systems), Section 1 Your name: KEY

Quiz 4

October 6, 2010

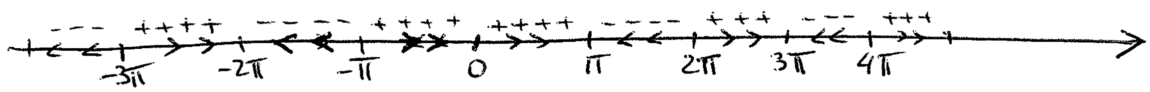
1. Consider a differential equation

$$\dot{x} = x \sin x.$$

- Find all fixed points of this equation

$X \sin X = 0 \iff x = \pi k$, k is an integer

- Draw a phase portrait



- For each fixed point determine its stability type (attracting, repelling or semiattracting)

0 is semiattracting

$2\pi k$ is repelling if $k > 0$ and attracting if $k < 0$

$(2k+1)\pi$ is attracting if $k \geq 0$ and repelling if $k < 0$

2. Consider a nonlinear system with a fixed point $\mathbf{x}^* = (1, 1)$

$$\dot{x} = -x^2 + y,$$
$$\dot{y} = x + y^2 - 2.$$

- Compute the linearized system at $\mathbf{x}^*$

$DF_{\bar{X}} = \begin{bmatrix} -2x & 1 \\ 1 & 2y \end{bmatrix}$; $DF_{\bar{X}^*} = \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix}$

Linearized system is

$$\dot{\bar{X}} = \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix} \bar{X}$$

- Determine stability type of $\mathbf{x}^*$ and type of the linearized system at $\mathbf{x}^*$

For matrix $\begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix}$ we have $\Delta = -4 - 1 = -5 < 0$, so the fixed point $\bar{X}^*$ is unstable and the linearized system at $\bar{X}^*$ is a saddle.