Math 372 (Dynamical Systems), Section 1 Your name: KEY

Quiz 3

September 29, 2010

1. Consider a differential equation

$$\dot{x} = 2(x+1).$$

- Find all fixed points of this equation

We solve $2(x+1)=0$, so $x=-1$ is the only fixed point

- Find the flow $\phi(t, x_0)$

$$\frac{dx}{dt} = 2(x+1)$$

$$\int \frac{dx}{x+1} = \int 2dt \Rightarrow \ln|x+1| = 2t + C \Rightarrow x = -1 + Ce^{2t}$$

$x_0 = x(0) = -1 + C \Rightarrow C = x_0 + 1$. Thus the flow:

$$\varphi(t; x_0) = -1 + (x_0+1)e^{2t}$$

- Determine the maximal interval for which $\phi(t, x_0)$ is defined

$\varphi(t, x_0)$ is defined for all t, so it is $(-\infty, \infty)$ (unless $x_0 = -1$)

2. Consider a linear system $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$, where the matrix

$$A = \begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix},$$

- Determine stability type of the origin (stable/unstable).

$$\left.\begin{aligned} &\tau = \operatorname{tr} A = 1 - 3 = -2 < 0 \\ &\Delta = \det A = 1\cdot(-3) + 2\cdot(-2) = 1 > 0 \\ &\tau^2 - 4\Delta = 4 - 4 = 0 \end{aligned}\right\} \Rightarrow \bar{0} \text{ is a degenerate stable node.}$$

In particular, it is asymptotically stable.

- Determine the type of a linear system.

- Describe the stable manifold (or basin of attraction) $W^s(\mathbf{0})$ of the origin.

Since every trajectory in degenerate stable node system is attracted to $\bar{0}$,

$W^s(\bar{0}) = \mathbb{R}^2$