Math 372 (Dynamical Systems), Section 1 Your name: KEY

Quiz 1

September 14, 2010

1. Solve the system of differential equations

$$\begin{cases} x_1' = x_1 + 8x_2 \\ x_2' = x_1 - x_2 \end{cases}$$

and sketch the phase portrait indicating the directions of the trajectories.

Corresponding matrix is:

$$A = \begin{bmatrix} 1 & 8 \\ 1 & -1 \end{bmatrix}$$

1) Find eigenvalues:

$$\det(A - \lambda I) = \det \begin{bmatrix} 1-\lambda & 8 \\ 1 & -1-\lambda \end{bmatrix} = (1-\lambda)(-1-\lambda) - 8 = \lambda^2 - 9 = (\lambda - 3)(\lambda + 3)$$

So $\lambda_1 = 3$, $\lambda_2 = -3$ are eigenvalues.

2) Find eigenvectors:

- For $\lambda_1 = 3$:

$$(A - 3I)\bar{v} = \begin{bmatrix} -2 & 8 \\ 1 & -4 \end{bmatrix} \bar{v} = \bar{0} \Leftrightarrow \begin{bmatrix} -2 & 8 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \bar{0} \Leftrightarrow -2v_1 + 8v_2 = 0.$$

So $\bar{v}^1 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ is an eigenvector for $\lambda_1 = 3$

- For $\lambda = -3$

$$(A + 3I)\bar{v} = \begin{bmatrix} 4 & 8 \\ 1 & 2 \end{bmatrix} \bar{v} = \bar{0} \Leftrightarrow 4v_1 + 8v_2 = 0.$$ So $\bar{v}^2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ is an eigenvector for $\lambda_2 = -3$

The general solution is

$$\bar{X}(t) = c_1 e^{3t} \begin{bmatrix} 4 \\ 1 \end{bmatrix} + c_2 e^{-3t} \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$$

Phase portrait: