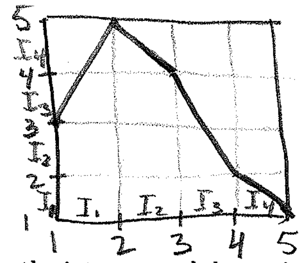
Math 372 (Dynamical Systems), Section 1 Your name: KEY

Quiz 10

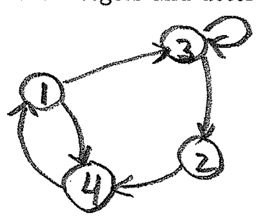
December 7, 2010

1. Let f be a continuous function defined on the interval $[1,5]$ in such a way that $f(1)=3$, $f(2)=5$, $f(3)=4$, $f(4)=2$, $f(5)=1$, and f is linear on intervals $[i,i+1]$, $i=1,2,3,4$.

(a) Sketch the graph of f.



(b) Label the intervals between the integers and determine the transition graph.



(c) For which n there is a period-n orbit? Determine a symbol sequence in terms of the intervals that shows each period that exists.

n	symbol sequence
1	$333\ldots = 3^\infty$
2	$(14)^\infty$
3	
4	$(3241)^\infty$
$n \geq 5$	$(3^{n-3}241)^\infty$

So there are points of all periods except 3.

2. Let $D(x) = 2x (\text{mod } 1)$ be the doubling map and let $x_0 = \frac{1}{5}$.

(a) Give a binary expansion of x_0.

$$\frac{1}{5} \to \frac{2}{5} \to \frac{4}{5} \to \frac{3}{5}$$

$$\frac{1}{5} < \frac{1}{2}, \quad \frac{2}{5} < \frac{1}{2}, \quad \frac{4}{5} > \frac{1}{2}, \quad \frac{3}{5} > \frac{1}{2} \Rightarrow \frac{1}{5} = 0.(0011)^\infty_2$$

(b) Determine whether $D^{10}(x_0) > \frac{1}{2}$ or $D^{10}(x_0) < \frac{1}{2}$.

The 11-th digit in the binary expansion of $\frac{1}{5} = 0.0011\,0011\,00\boxed{1}\ldots$ is 1,

so $D^{10}(\frac{1}{5}) \in [\frac{1}{2}, 1]$. It also can't be $\frac{1}{2}$,

thus $D^{10}(\frac{1}{5}) > \frac{1}{2}$