Math 304 (Linear Algebra), Section 2 Your name: KEY

Quiz 9

November 12, 2010

1. Let P_2 be the space of polynomials of x of degree at most 2. Let $X = (1, x, x^2)$ be a basis for P_2. Suppose $D: P_2 \to P_2$ is a linear transformation defined by

$$D(f(x)) = f'(x) + f(x).$$

Find the matrix ${}_X D_X$ of transformation D with respect to basis X.

$${}_X D_X = \left[K_X(D(1)) \,\middle|\, K_X(D(x)) \,\middle|\, K_X(D(x^2)) \right] =$$

$$= \left[K_X(0+1) \,\middle|\, K_X(1+x) \,\middle|\, K_X(2x+x^2) \right] = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

2. Let $F: \mathbb{R}^2 \to \mathbb{R}^2$ be given by the matrix (with respect to the standard basis E_2)

$${}_{E_2}F_{E_2} = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$$

Find matrix ${}_Y F_Y$ of F with respect to the basis $Y = \left(\begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right)$

First we find ${}_{E_2}I_Y$ and ${}_Y I_{E_2}$.

$${}_{E_2}I_Y = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}. \quad {}_Y I_{E_2} = \left({}_{E_2}I_Y\right)^{-1}$$

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ -2 & -1 & 0 & 1 \end{array}\right] \xrightarrow{r_2 \to r_2 + 2r_1} \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{array}\right] \xrightarrow{r_1 \to r_1 - r_2} \left[\begin{array}{cc|cc} 1 & 0 & -1 & -1 \\ 0 & 1 & 2 & 1 \end{array}\right].$$

So ${}_Y I_{E_2} = \begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$.

Therefore

$${}_Y F_Y = \left({}_Y I_{E_2}\right)\left({}_{E_2}F_{E_2}\right)\left({}_{E_2}I_Y\right) = \begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 0 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 7 & 4 \\ -10 & -5 \end{bmatrix}$$