Math 304 (Linear Algebra), Section 2 Your name:

Quiz 8

November 5, 2010

1. Let $Pol(2)$ denote the space of all polynomials of x of degree up to 2.

(a) What is the basis for $Pol(2)$?

$(1, x, x^2)$

(b) What is dimension of $Pol(2)$ (just state, do not prove)?

$3 = \#$ of elements in the basis

(c) Prove that $f(x) = x^2 - x + 1 \in Pol(2)$ and $g(x) = x^2 + x + 1 \in Pol(2)$ are linearly independent.

Let $T: Pol(2) \to \mathbb{R}^3$ be an isomorphism defined by $T(a_0 + a_1 x + a_2 x^2) = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$. Then $\{f, g\}$ is lin. independent in $Pol(2) \Leftrightarrow \{T(f), T(g)\}$ is linearly indep. in $\mathbb{R}^3$.

$$\left[T(f) \middle| T(g)\right] = \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 0 & 0 \end{bmatrix}.$$

rank $= 2 = \#$ of columns, so $\{f, g\}$ is lin. indep.

(d) Express $h(x) = x^2 - 5x + 1 \in Pol(2)$ as a linear combination of $f(x)$ and $g(x)$.

We first express $T(h)$ as a linear combination of $T(f)$ and $T(g)$.

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ -1 & 1 & -5 \\ 1 & 1 & 1 \end{array}\right] \xrightarrow[r_3 \to r_3 - r_1]{r_2 \to r_2 + r_1} \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 2 & -4 \\ 0 & 0 & 0 \end{array}\right] \xrightarrow[2) r_1 \to r_1 - r_2]{1) r_2 \to r_2/2} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{array}\right]$$

$\uparrow$ $T(h)$

Therefore, $\boxed{h(x) = 3f(x) - 2g(x)}$