Math 304 (Linear Algebra), Section 2 Your name: KEY

Quiz 6

October 22, 2010

1. Let T be a transformation from $\mathbb{R}^4$ to $\mathbb{R}^2$ defined by a matrix

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 1 & 2 & 1 & -1 \end{bmatrix}$$

- Find the basis for the column space Col(A) of A (same as image Im(T) of T)

Reduce A to REF

$A \xrightarrow{r_2 \to r_2 - r_1} \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$. Pivot columns in A are 1-st and 3-rd. So the basis for Col(A) is

$$\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

- Find the basis for the null space Null(A) of A (same as kernel Ker(T) of T)

A is already reduced to RREF

General solution is

$$\begin{aligned} x_1 &= -2x_2 - 1 \cdot x_4, \\ x_2 &= 1 \cdot x_2 + 0 \cdot x_4, \\ x_3 &= 0 x_2 + 2 \cdot x_4, \\ x_4 &= 0 \cdot x_2 + 1 \cdot x_4. \end{aligned}$$

Therefore, the basis for Ker T is

$$\left(\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 2 \\ 1 \end{bmatrix} \right)$$

- Find the basis for the row space Row(A) of A

The basis for Row(A) consists of nonzero rows in REF(A). Thus, the basis here is

$$\left(\begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ -2 \end{bmatrix} \right).$$