Math 304 (Linear Algebra), Section 2 Your name: KEY

Quiz 5

October 15, 2010

1. Complete the following definition: The set X of vectors in $\mathbb{R}^n$ is called *linearly dependent* if

there is a linear combination

$$c_1\overline{X}_1 + \dots + c_n\overline{X}_n = \overline{0}$$

of different vectors in X, which is equal to $\overline{0}$ and such that not all the coefficients are zeros.

2. Let

$$X = \left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ -4 \end{bmatrix} \right\}.$$

Determine if X is linearly dependent or independent. If X is linearly dependent, find linear dependence relation for X.

Consider the matrix corresponding to X

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & -1 \\ -1 & 2 & -4 \end{bmatrix} \xrightarrow{r_3 \to r_3 + r_1} \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix} \xrightarrow{r_3 \to r_3 - r_2} \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

So $\operatorname{rank} A = 2 < 3 = \#$ of vectors in X

Therefore, X is linearly dependent

To find linear dependence relation reduce A to RREF

$$\xrightarrow{r_1 \to r_1 + r_2} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{aligned} c_1 &= -2c_3 \\ c_2 &= c_3 \\ c_3 &= \text{anything} \end{aligned}$$

Pick $c_3 = 1$. Then $c_1 = -2$, $c_2 = 1$, and linear dep. relation is

$$-2\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + 1\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} + 1\begin{bmatrix} 3 \\ -1 \\ -4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$