Math 304 (Linear Algebra), Section 2 Your name: KEY

Quiz 4

October 8, 2010

1. Complete the following definitions

(a) A span of a collection of vectors $\{v_1, v_2, \dots, v_n\}$ is the set of all linear combinations of $\bar{v}_1, \dots, \bar{v}_n$.

$\text{span}\{\bar{v}_1, \dots, \bar{v}_n\} = \{c_1\bar{v}_1 + \dots + c_n\bar{v}_n : c_i \in \mathbb{R}\}$

(b) A transformation $F: \mathbb{R}^n \to \mathbb{R}^m$ is called linear if for all $\bar{u}, \bar{v} \in \mathbb{R}^n$ and each $c \in \mathbb{R}$:

1) $F(\bar{u}+\bar{v}) = F(\bar{u}) + F(\bar{v})$

2) $F(c\bar{u}) = cF(\bar{u})$

2. Let $T: \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Recall that the kernel $\ker T$ of T is the set of all vectors $\mathbf{x} \in \mathbb{R}^n$ such that $T(\mathbf{x}) = \mathbf{0}$. Show that $\ker T$ is a subspace of $\mathbb{R}^n$.

We need to check 3 conditions

1) $\bar{0} \in \ker T$ since $T(\bar{0}) = \bar{0}$ ✓

2) Let $\bar{u}, \bar{v} \in \ker T$. Then $T(\bar{u}) = T(\bar{v}) = \bar{0}$.

Therefore $T(\bar{u}+\bar{v}) \underset{T \text{ is linear.}}{=} T(\bar{u}) + T(\bar{v}) = \bar{0} + \bar{0} = \bar{0}$

Thus $\bar{u}+\bar{v} \in \ker T$ ✓

3) Let $\bar{u} \in \ker T$ and $c \in \mathbb{R}$. Then

$T(c\bar{u}) \underset{T \text{ is linear}}{=} cT(\bar{u}) \underset{\bar{u} \in \ker T}{=} c \cdot \bar{0} = \bar{0}$. ✓

So $c\bar{u} \in \ker T$.

Hence, $\ker T$ is a subspace of $\mathbb{R}^n$.