Math 304 (Linear Algebra), Section 2 Your name: KEY

Quiz 3

September 27, 2010

1. Let T be a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$ defined by:

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ x_2 - x_1 \\ x_2 + 2x_1 \end{bmatrix}.$$

(a) What is the domain of T? $\mathbb{R}^2$

(b) What is the codomain of T? $\mathbb{R}^3$

(c) Find the matrix corresponding to T.

$$\left[T(\bar{e}_1) \,\middle|\, T(\bar{e}_2) \right] = \left[T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) \,\middle|\, T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) \right] = \left[\begin{array}{c|c} 1+0 & 0+1 \\ 0-1 & 1-0 \\ 0+2\cdot 1 & 1+2\cdot 0 \end{array}\right] = \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 2 & 1 \end{bmatrix}$$

(d) Find the vector in $\mathbb{R}^3$ which is not in the image of T (thus proving that T is not onto).

We need to find a vector $\bar{b} \in \mathbb{R}^3$ that makes $T(\bar{x}) = \bar{b}$ inconsistent.

First, leave column blank.

$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 2 & 1 & 1 \end{array}\right] \xrightarrow[r_3 \to r_3 - 2r_1]{r_2 \to r_2 + r_1} \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & -1 & 1 \end{array}\right] \xrightarrow{r_3 \to r_3 + \frac{1}{2} r_2} \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{array}\right]$$

pick a vector which makes a system inconsistent

reverse row operations

Thus $\bar{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ is not in the image of T.