

Name: KEY

1) Given the following system of linear equations:

$$x + y + z + t = 2$$

$$y + z + t = 1$$

$$2x + 2y + 3z + 3t = 3$$

$$x + y + z - t = 4$$

a) Determine the augmented matrix for the above system and identify the coefficient matrix.

Augmented matrix:
$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 & 1 \\ 2 & 2 & 3 & 3 & 3 \\ 1 & 1 & 1 & -1 & 4 \end{array} \right]$$

Coefficient matrix:
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 2 & 2 & 3 & 3 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$

b) Bring the above augmented matrix into reduced row echelon form and indicate each elementary row operation.

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 & 1 \\ 2 & 2 & 3 & 3 & 3 \\ 1 & 1 & 1 & -1 & 4 \end{array} \right] \xrightarrow{\substack{r_3 \rightarrow r_3 - 2r_1 \\ r_4 \rightarrow r_4 - r_1}} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & -2 & 2 \end{array} \right] \xrightarrow{r_4 \rightarrow -\frac{1}{2}r_4} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{array} \right]$$

REF

$$\begin{array}{l} r_3 \rightarrow r_3 - r_4 \\ r_2 \rightarrow r_2 - r_4 \\ r_1 \rightarrow r_1 - r_4 \end{array} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 3 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array} \right] \xrightarrow{\substack{r_2 \rightarrow r_2 - r_3 \\ r_1 \rightarrow r_1 - r_3}} \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array} \right] \xrightarrow{r_1 \rightarrow r_1 - r_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array} \right]$$

RREF

c) Determine the solution set of the system using the reduced row echelon form and check your result.

The solution is:

$$\begin{aligned} x &= 1 \\ y &= 2 \\ z &= 0 \\ t &= -1 \end{aligned}$$

Check:

$$\begin{aligned} 1 + 2 + 0 - 1 &= 2 \quad \checkmark \\ 2 + 0 - 1 &= 1 \quad \checkmark \\ 2 + 4 + 0 - 3 &= 3 \quad \checkmark \\ 1 + 2 + 0 + 1 &= 4 \quad \checkmark \end{aligned}$$