Math 304 (Linear Algebra), Section 2 Your name: KEY

Quiz 12

December 8, 2010

1. Let $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} -2 \\ 1 \\ 3 \\ 1 \end{bmatrix}$. Compute

(a) $\mathbf{v} \cdot \mathbf{w} = 1\cdot(-2) + 0\cdot 1 + 2\cdot 3 + (-1)\cdot 1 = 3$

(b) $\|\mathbf{v}\| = \sqrt{1^2 + 0^2 + 2^2 + (-1)^2} = \sqrt{6}$

(c) a unit vector $\mathbf{u}$ that is a scalar multiple of a vector $\mathbf{v}$

$$\bar{u} = \frac{\bar{v}}{\|\bar{v}\|} = \frac{1}{\sqrt{6}}\bar{v} = \begin{bmatrix} \frac{1}{\sqrt{6}} \\ 0 \\ \frac{2}{\sqrt{6}} \\ -\frac{1}{\sqrt{6}} \end{bmatrix}$$

(d) a basis for $\text{Span}\{\mathbf{v}, \mathbf{w}\}^{\perp}$

$$\text{Span}\{\bar{v}, \bar{w}\}^{\perp} = \text{Null}\left(\begin{bmatrix} 1 & 0 & 2 & -1 \\ -2 & 1 & 3 & 1 \end{bmatrix}\right)$$

$$\begin{bmatrix} 1 & 0 & 2 & -1 \\ -2 & 1 & 3 & 1 \end{bmatrix} \xrightarrow{r_2 \to r_2 + 2r_1} \begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & 7 & -1 \end{bmatrix}$$

$$\begin{aligned} x_1 &= -2x_3 + 1x_4 \\ x_2 &= -7x_3 + 1x_4 \\ x_3 &= 1\cdot x_3 + 0x_4 \\ x_4 &= 0\cdot x_3 + 1\cdot x_4 \end{aligned}$$

So the basis for $\text{Span}(\bar{v}, \bar{w})^{\perp}$ is

$$\left(\begin{bmatrix} -2 \\ -7 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}\right)$$