

Quiz 7

April 19, 2010

1. Prove that for any $x, y \in \mathbb{R}$

$$|xy| = |x| \cdot |y|$$

There are 4 possible cases:

- 1) $x \geq 0, y \geq 0$. Then $xy \geq 0$ and $|xy| = xy = |x| \cdot |y|$,
- 2) $x \geq 0, y < 0$. Then $xy \leq 0$ and $|xy| = -xy = x \cdot (-y) = |x| \cdot |y|$,
- 3) $x < 0, y \geq 0$. Then $xy \leq 0$ and $|xy| = -xy = (-x)y = |x| \cdot |y|$,
- 4) $x < 0, y < 0$. Then $xy > 0$ and $|xy| = xy = (-x)(-y) = |x| \cdot |y|$.

So in all possible cases we have $|xy| = |x| \cdot |y|$.

2. Prove using the definition of the limit that

$$\lim_{k \rightarrow \infty} \frac{k-1}{k} = 1.$$

$\forall \epsilon > 0 \exists N \in \mathbb{N}$ s.t. $N > \frac{1}{\epsilon}$. Then $\forall k \geq N$

$$\left| \frac{k-1}{k} - 1 \right| = \left| -\frac{1}{k} \right| = \frac{1}{k} \leq \frac{1}{N} < \epsilon.$$

Thus $\lim_{k \rightarrow \infty} \frac{k-1}{k} = 1$.