Math 330 (Number Systems), Section 3 Your name: KEY

# Quiz 3

February 19, 2010

1. Rewrite the following statements using quantifiers

   (a) For each natural number $n$ such that 2 divides $n$ the equation $x^2 + y^2 = n$ has an integer solution.

   $$\forall n \in \mathbb{N} \left[ (2 | n) \Rightarrow \left( \exists x, y \in \mathbb{Z} \text{ s.t. } x^2 + y^2 = n \right) \right]$$

   (b) There is no natural number $n$ such that for any integer $m$ the equalities $n^2 = -m$ and $(-n)^2 = m^2$ hold.

   $$\overline{\exists n \in \mathbb{N} \left( \forall m \in \mathbb{Z} : (n^2 = -m) \wedge ((-n)^2 = m^2) \right)}$$

   or

   $$\forall n \in \mathbb{N} \left[ \exists m \in \mathbb{Z} : (n^2 \neq -m) \vee ((-n)^2 \neq m^2) \right]$$

2. Prove that the statement $A \Rightarrow (B \Rightarrow A)$ is a tautology (always true no matter what logical values for $A$ and $B$ you plug).

Let's construct a truth table

| $A$ | $B$ | $B \Rightarrow A$ | $A \Rightarrow (B \Rightarrow A)$ |
|---|---|---|---|
| T | T | T | T |
| T | F | T | T |
| F | T | F | T |
| F | F | T | T |

Therefore $A \Rightarrow (B \Rightarrow A)$ is a tautology.