Math 330 (Number Systems), Section 3 (Dmytro Savchuk) Spring 2010 Homework assignment 8 (due Monday, March 15)

This is a preparational homework for the midterm. It contains 2 parts. The first part is over equivalence relation and integers modulo n. This part will be graded. The second part contains problems that are supposed to prepare you for the midterm. I will not grade the second part but will be glad to answer any questions about the problems in this part. Be sure that you know how to solve these problems.

Part I (to be graded)

- 1. Do Project 6.7 (all items)
- 2. Do Project 6.8 (ii)
- 3. Prove Propositions 6.17, 6.25
- 4. Prove that there are infinitely many prime numbers (Hint: if there were only finitely many prime numbers, say p_1, p_2, \ldots, p_s , then $n = p_1 p_2 \cdots p_s + 1$ should contradict Proposition 6.25)

Part II (not to be graded, but you should, of course, do it)

- 1. Let $f: \mathbb{Z} \to \mathbb{Z}$ be a function defined by $f(n) = n^2$.
 - Prove that $f(\mathbb{Z}) = f(\mathbb{N}) \cup \{0\}.$
 - Find $f^{-1}(\{9\})$.
 - Find $f(\{-2,2\})$.
- 2. Prove that for each $n \in \mathbb{N}$ either $4|n^2$ or $4|(n^2-1)$.
- 3. Find all $n \in \mathbb{N}$ such that $2^n < n!$
- 4. Prove that the following statement is a tautology $A \Rightarrow ((A \equiv B) \equiv B)$
- 5. Negate the following sentences
 - If for each integer n
 - For every $x \in \mathbb{Z}$ there is a subset A of \mathbb{Z} whose smallest element is x
 - If the derivative of a function at some point x_0 is 0 then x_0 is a local extremum
 - x is an element of $A \cap B$
 - x is an element of $A \cup B$
 - Every person in the world is the friend of someone else
 - Every triangle has 3 sides
- 6. Prove that for sets A and B the sets A B, B A and $A \cap B$ form a partition of $A \cup B$.
- 7. Do Project 5.12
- 8. Prove Proposition 2.17(iii)

9. Prove that for
$$n \in \mathbb{N}$$
 we have $\sum_{j=0}^{n} (-1)^{j} \binom{n}{j} = 0$.