This is a preparational homework for the midterm. It contains 2 parts. The first part is over equivalence relation and integers modulo $n$. This part will be graded. The second part contains problems that are supposed to prepare you for the midterm. I will not grade the second part but will be glad to answer any questions about the problems in this part. Be sure that you know how to solve these problems.

## Part I (to be graded)

1. Do Project 6.7 (all items)
2. Do Project 6.8 (ii)
3. Prove Propositions 6.17, 6.25
4. Prove that there are infinitely many prime numbers (Hint: if there were only finitely many prime numbers, say $p_{1}, p_{2}, \ldots, p_{s}$, then $n=p_{1} p_{2} \cdots p_{s}+1$ should contradict Proposition 6.25)

## Part II (not to be graded, but you should, of course, do it)

1. Let $f: \mathbb{Z} \rightarrow \mathbb{Z}$ be a function defined by $f(n)=n^{2}$.

- Prove that $f(\mathbb{Z})=f(\mathbb{N}) \cup\{0\}$.
- Find $f^{-1}(\{9\})$.
- Find $f(\{-2,2\})$.

2. Prove that for each $n \in \mathbb{N}$ either $4 \mid n^{2}$ or $4 \mid\left(n^{2}-1\right)$.
3. Find all $n \in \mathbb{N}$ such that $2^{n}<n$ !
4. Prove that the following statement is a tautology $A \Rightarrow((A \equiv B) \equiv B)$
5. Negate the following sentences

- If for each integer $n$
- For every $x \in \mathbb{Z}$ there is a subset $A$ of $\mathbb{Z}$ whose smallest element is $x$
- If the derivative of a function at some point $x_{0}$ is 0 then $x_{0}$ is a local extremum
- $x$ is an element of $A \cap B$
- $x$ is an element of $A \cup B$
- Every person in the world is the friend of someone else
- Every triangle has 3 sides

6. Prove that for sets $A$ and $B$ the sets $A-B, B-A$ and $A \cap B$ form a partition of $A \cup B$.
7. Do Project 5.12
8. Prove Proposition 2.17(iii)
9. Prove that for $n \in \mathbb{N}$ we have $\sum_{j=0}^{n}(-1)^{j}\binom{n}{j}=0$.
