Math 330 (Number Systems), Section 3 (Dmytro Savchuk) Spring 2010

Homework assignment 13

This is the first part of a preparational homework for the final. I will post another part soon. I will not collect this homework, but I encourage you to solve these problems before final.

1. Let $(x_n)_{n\in\mathbb{N}}$ be a sequence defined recursively by $x_1 = \sqrt{2}$ and $x_{n+1} = \sqrt{2 + x_n}$ for $n \geq 1$ (i.e. $x_n = \sqrt{2 + \sqrt{2 + \sqrt{\cdots + \sqrt{2}}}}$ with n square roots in the last expression.
   (a) Prove that $(x_n)_{n\in\mathbb{N}}$ is increasing;
   (b) Prove that $(x_n)_{n\in\mathbb{N}}$ is bounded;
   (c) Conclude that $(x_n)_{n\in\mathbb{N}}$ is convergent and find $\lim_{n\to\infty} x_n$.
2. Let $f : A \to B$ be a function and let U be a subset of A. Define $f|_U : U \to f(U)$ by $\tilde{f}(x) = f(x)$ for each $x \in U$. This function $\tilde{f}$ is called a *restriction* of f onto U.
   (a) Observe that $f|_U$ is well defined (i.e. $f|_U(x) \in f(U)$ for all $x \in U$).
   (b) Prove that if f is injection then so is $f|_U$.
   (c) Prove that if f is surjection then so is $f|_U$.
   (d) Conclude that if f is bijection then so is $f|_U$.
3. Prove proposition 13.10 (you may want to use Proposition 13.9 – be sure to understand the proof).
4. Prove proposition 13.19.