Math 330 (Number Systems), Section 3 (Dmytro Savchuk) Spring 2010

Homework assignment 12 (due Wednesday, April 28)

1. Prove the Squeeze theorem: let $(a_n)_{n\in\mathbb{N}}$, $(b_n)_{n\in\mathbb{N}}$ be convergent sequences such that $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = L$ and assume that terms of a sequence $(c_n)_{n\in\mathbb{N}}$ satisfy $a_n \leq c_n \leq b_n$ for all $n \in \mathbb{N}$. Then $\lim_{n\to\infty} c_n = L$.
2. Complete the second proof of Proposition 10.17 that we have started in class. Recall that we proved $\lim_{n\to\infty} |x|^n = 0$ for $|x| < 1$, so we need to prove that this implies $\lim_{n\to\infty} x^n = 0$.
3. Do Project 10.23
4. Compute the limits of the sequences below if they exist. Justify all your claims.
   (a) $a_n = \dfrac{1}{\sqrt{n}}$,
   (b) $b_n = -n^2$,
   (c) $c_n = \dfrac{1}{n^2} + x^{2n} \cdot \dfrac{n}{n+1}$, where $|x| < 1$.
5. Prove that if a sequence $(a_n)_{n\in\mathbb{N}}$ is convergent and a sequence $(b_n)_{n\in\mathbb{N}}$ differs from $(a_n)_{n\in\mathbb{N}}$ in only finitely many terms, then
$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} a_n.$$
   (This means that arbitrary large beginnings of a sequence do not affect its limit).