Math 371 (Differential Equations), Section 3 Your name: KEY

Quiz 8

November 24, 2009

1. Calculate the Laplace Transform of

$$f(t) = \begin{cases} 0, & t < 1 \\ t, & t \geq 1 \end{cases}$$

$$f(t) = t \cdot u(t-1)$$

$$\mathcal{L}\{f\} = \mathcal{L}\{((t-1)+1)u(t-1)\} = e^{-s} \cdot \mathcal{L}\{t+1\} = e^{-s} \cdot \left(\frac{1}{s^2} + \frac{1}{s}\right) =$$

$$= e^{-s} \cdot \frac{s+1}{s^2}$$

2. Using the answer from problem 1 solve the following differential equation

Let $Y = \mathcal{L}\{y\}$. Then $y'' + y = f(t), \quad y(0) = 0, \quad y'(0) = 1$

$$\mathcal{L}\{y''\} = s^2 Y - s y(0) - y'(0) = s^2 Y - 1$$

$$\mathcal{L}\{y'' + y\} = s^2 \cdot Y - 1 + Y = Y(s^2+1) - 1 = \mathcal{L}\{f(t)\}$$

$$Y(s^2+1) - 1 = e^{-s} \cdot \frac{s+1}{s^2}$$

$$Y = \underbrace{\frac{1}{s^2+1}}_{\mathrm{I}} + \underbrace{e^{-s} \cdot \frac{s+1}{(s^2+1)s^2}}_{\mathrm{II}}$$. Now $y = \mathcal{L}^{-1}\{Y\}$

I. $\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} = \sin t$

II. $$\frac{s+1}{(s^2+1)s^2} = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs+D}{s^2+1} = \frac{As(s^2+1) + B(s^2+1) + (Cs+D)s^2}{''} = \frac{(A+C)s^3 + (B+D)s^2 + As + B}{''}$$

$A+C=0$, $B+D=0$, $A=1$, $B=1$ $\Rightarrow$ $C=-1$, $D=-1$

$$\mathcal{L}^{-1}\left\{\frac{s+1}{(s^2+1)s^2}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s} + \frac{1}{s^2} - \frac{s}{s^2+1} - \frac{1}{s^2+1}\right\} = 1 + t - \cos t - \sin t$$

$$y = \sin t + \mathcal{L}^{-1}\left\{e^{-s} \cdot \frac{s+1}{(s^2+1)s^2}\right\} = \sin t + (\cancel{1} + (t-\cancel{1}) - \cos(t-1) - \sin(t-1))u(t-1) =$$

$$= \sin t + (t - \cos(t-1) - \sin(t-1))u(t-1)$$