Math 371 (Differential Equations), Section 3 Your name: KEY

Quiz 7

November 17, 2009

1. Solve the following differential equation using power series method

$$y'' + 2x^2y' - 2xy = 0$$

Suppose $y(x) = a_0 + a_1x + a_2x^2 + \dots$ is a solution.

Then
$$y'(x) = a_1 + 2a_2x + 3a_3x^2 + \dots$$
$$y''(x) = 2a_2 + 3\cdot 2a_3x + 4\cdot 3a_4x^2 + \dots + a_{n+2}(n+2)(n+1)x^n + \dots$$
$$2x^2y'(x) = 0 + 0 + 2a_1x^2 + \dots + 2(n-1)a_{n-1}x^n + \dots$$
$$-2xy(x) = 0 - 2a_0x - 2a_1x^2 - \dots - 2a_{n-1}x^n - \dots$$

So $2a_2 + 0 + 0 = 0$

@x: $3\cdot 2a_3 + 0 - 2a_0 = 0$

@x^2: $4\cdot 3a_4 + 2a_1 - 2a_1 = 0$

@x^n: $(n+2)(n+1)a_{n+2} + 2(n-1)a_{n-1} - 2a_{n-1} = 0$

or

$$\begin{cases} a_0, a_1 - \text{arbitrary constants} \\ a_2 = 0 \\ a_3 = \frac{a_0}{3} \\ a_{n+2} = -\frac{2(n-2)}{(n+2)(n+1)}a_{n-1} \end{cases}$$

2. Calculate the Laplace Transform of $f(t) = e^{3t}cos(2t)$.

$$\mathcal{L}\{f\} = \mathcal{L}\{\cos 2t\}(s-3) = \frac{s-3}{(s-3)^2+4}$$

3. Calculate the Inverse Laplace Transform of $F(s) = \frac{1}{s-3} + \frac{6}{s^4}$.

$$\mathcal{L}^{-1}\{F\} = \mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\} + \mathcal{L}^{-1}\left(\frac{6}{s^4}\right) = e^{3t} + t^3$$