Math 371 (Differential Equations), Section 3 Your name: KEY

Quiz 6

November 5, 2009

1. Solve the following differential equation

$$x^2y'' + 2xy' - 2y = 0$$

This is Euler-Cauchy equation. $a=2$, $b=-2$.
The corresponding equation for m is

$m^2+(a-1)m+b=0$

$m^2+m-2=0$

$(m+2)(m-1)=0 \Rightarrow m=-2$ or $m=1$. Thus the general solution is $y=C_1x^{-2}+C_2x$

2. Solve the system of differential equations

$$\begin{cases} y_1' = y_1 + 8y_2 \\ y_2' = y_1 - y_2 \end{cases}$$

Corresponding matrix is

$A=\begin{bmatrix} 1 & 8 \\ 1 & -1 \end{bmatrix}$.

1) Find eigenvalues:

$\det(A-\lambda I) = \det\begin{bmatrix} 1-\lambda & 8 \\ 1 & -1-\lambda \end{bmatrix} = (1-\lambda)(-1-\lambda)-8 = \lambda^2-9 = (\lambda-3)(\lambda+3)$.

So $\lambda_1=3$, $\lambda_2=-3$ are eigenvalues

2) Find eigenvectors

- For $\lambda_1=3$

$(A-3I)\bar{v} = \begin{bmatrix} -2 & 8 \\ 1 & -4 \end{bmatrix}\bar{v} = \bar{0} \Leftrightarrow \begin{bmatrix} -2 & 8 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0 \Leftrightarrow -2v_1+8v_2=0$

Pick $v_1=4$, $v_2=1 \Rightarrow \bar{v}_1=\begin{bmatrix} 4 \\ 1 \end{bmatrix}$ is an eigenvector for $\lambda_1=3$

- For $\lambda_1=-3$

$(A+3I)\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0 \Leftrightarrow 4v_1+8v_2=0$. Pick $v_1=2$, $v_2=-1$.

So $\bar{v}_2=\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ is an eigenvector for $\lambda_2=-3$.

Solution is $\bar{y}=c_1e^{3x}\begin{bmatrix} 4 \\ 1 \end{bmatrix}+c_2e^{-3x}\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ or $\begin{matrix} y_1=c_1\cdot 4e^{3x}+c_2 2e^{-3x} \\ y_2=c_1e^{3x}-c_2e^{-3x} \end{matrix}$