Math 371 (Differential Equations), Section 3 Your name: KEY

Quiz 5

October 13, 2009

1. Solve the following differential equation

$$y'' - 4y' + 5y = 0$$

Characteristic polynomial is

$\lambda^2 - 4\lambda + 5 = (\lambda - 2 + i)(\lambda - 2 - i) \Rightarrow \lambda = 2 \pm i$

Therefore the general solution is

$y = c_1 e^{2x} \cos x + c_2 e^{2x} \sin x$

2. Use the method of differential operators to solve the following differential equation

$$y'' - 4y' + 4y = e^{3x}$$

We have

$(D^2 - 4D + 4)(y) = e^{3x}$

$(D-2)((D-2)(y)) = e^{3x}$

$\begin{cases} (D-2)(y) = u & (1) \\ (D-2)(u) = e^{3x} & (2) \end{cases}$

Solve (2): $u' - 2u = e^{3x}$

Integrating factor $I(x) = e^{\int(-2)dx} = e^{-2x}$

$(ue^{-2x})' = e^{3x} \cdot e^{-2x} = e^x$

$ue^{-2x} = \int e^x \, dx = e^x + C \Rightarrow u = e^{3x} + Ce^{2x}$. Plug it into (1)

$y' - 2y = e^{3x} + Ce^{2x}$

$I(x) = e^{\int(-2)dx} = e^{-2x}$

$(y \cdot e^{-2x})' = e^{3x} \cdot e^{-2x} + Ce^{2x} \cdot e^{-2x} = e^x + C$

$ye^{-2x} = \int (e^x + C)\, dx = e^x + Cx + D \Rightarrow \boxed{y = e^{3x} + Cxe^{2x} + De^{2x}}$