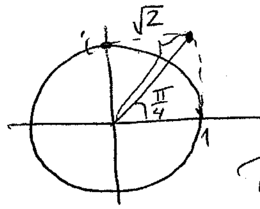
Math 371 (Differential Equations), Section 3 Your name: KEY

Quiz 4

October 1, 2009

1. Calculate $(1+i)^{50}$



We have

$$1+i=\sqrt{2}\cdot e^{i\frac{\pi}{4}}.$$

Therefore

$$(1+i)^{50}=\left(\sqrt{2}\cdot e^{i\frac{\pi}{4}}\right)^{50}=\sqrt{2}^{50}\cdot e^{i\cdot\frac{\pi}{4}\cdot 50}=2^{25}\cdot e^{\frac{25\pi}{2}i}=$$

$$=2^{25}\cdot e^{12i\pi+\frac{\pi}{2}i}=2^{25}\cdot\underbrace{\left(e^{2i\pi}\right)^{6}}_{=1}\cdot\underbrace{e^{\frac{i\pi}{2}}}_{=i}=\boxed{2^{25}\cdot i}$$

2. Solve the following linear differential equation

$$y''+2y'+y=0 \qquad (1)$$

if it is known that $y_1(x)=e^{-x}$ is one of its solutions. You can either find the second linearly independent solution or solve this equation as a linear differential equation with constant coefficients.

Solution 1: If $y_1(x)=e^{-x}$ is the solution, we look for $y_2(x)=u(x)e^{-x}$.

Plug $y_2(x)$ into (1):

$$(u(x)e^{-x})''+2(u(x)e^{-x})'+u(x)e^{-x}=0$$

$$u''(x)e^{-x}-2e^{-x}u'(x)+e^{-x}u(x)+2u'(x)e^{-x}-2u(x)e^{-x}+u(x)e^{-x}=0$$

$u''(x)=0$ sum up to 0 since $y_1(x)=e^{-x}$ is a solution.

$u'(x)=C$

$u(x)=Cx+D$. We pick $u(x)=x$.

Thus $y_2(x)=xe^{-x}$ and general solution is $\boxed{y=C_1e^{-x}+C_2xe^{-x}}$

Solution 2: This equation is linear with constant coefficients. Characteristic polynomial is $\lambda^2+2\lambda+1=0 \Leftrightarrow (\lambda+1)^2=0 \Leftrightarrow \lambda=-1$.

We have a double root $\Rightarrow$ solution is $\boxed{y=C_1e^{-x}+C_2xe^{-x}}$