

Quiz 3

September 22, 2009

1. Solve the following differential equation

$$y' = \tan\left(\frac{y}{x}\right) + \frac{y}{x}$$

Make a substitution $u = \frac{y}{x}$.

Then $y = ux$ and $y' = u'x + u$

$$u'x + u = \tan u + u$$

$$u'x = \tan u, \text{ check that } u=0 \text{ is a solution.}$$

$$\int \frac{du}{\tan u} = \int \frac{dx}{x}$$

$$\ln |\sin u| = \ln |x| + C$$

$\sin u = Cx$, C any real number

$$\boxed{\sin \frac{y}{x} = Cx}$$

2. Solve the following linear differential equation

$$y' + y \sin x = e^{\cos x}$$

This is a linear diff. equation in the standard form.

$$p(x) = \sin x, \quad q(x) = e^{\cos x}$$

① Find integrating factor:

$$I(x) = e^{\int p(x) dx} = e^{\int \sin x dx} = e^{-\cos x}$$

$$\textcircled{2} \quad (e^{-\cos x} \cdot y)' = e^{-\cos x} \cdot e^{\cos x} = e^0 = 1$$

$$e^{-\cos x} \cdot y = C$$

$$\boxed{y = C \cdot e^{\cos x}}$$